ECONOMIC ORDER QUANTITY (EOQ)/VARIABLE SAFETY LEVEL (VSL)

1. Introduction.

a. This unit of instruction will address the Economic Inventory Policies (EIP) employed by the U.S. Army Materiel Command (AMC). Current Army inventory managed by AMC exceeds 50,000 items. Manual management of such a large number of items is virtually impossible. Therefore, only the development of automated data processes used within the Department of Defense (DOD) has enabled the manager to function effectively. However, while the system has grown in complexity and size, the basic inventory decisions confronting the manager have remained constant. The two decisions required for each item in the inventory are:

(1) How much of an item to order.

(2) When to order replenishment of the item.

b. While the basic decisions have remained the same, the methods of arriving at the recommendations have changed. The use of automation has allowed for the introduction of complex models employing numerous variables and constraints. These models are easily solved by the computer but are prohibitive for manual calculations.

c. This unit of instruction will introduce you to some of the basic inventory models used by managers to determine how much and when to buy. Through the use of some of the more simple models that allow for manual calculations, you should get a feel for how the inventory system works.

d. The basis for an economic inventory system is established by DOD Instruction (DODI) 4140.39, Procurement Cycles and Safety Levels of Supply for Secondary Items. This is applicable to all DOD activities worldwide that perform inventory management functions that deal with EIP and safety levels. DOD 4140.1-R (Materiel Management Regulation) incorporates many DOD inventory reduction plans required by the DOD "builddown" process and was approved in January 1993. It establishes a policy for peacetime operation of supply on the most economical basis. These policies consider military necessity based on repetitive demands of secondary items. Excluded from this are ammunition, bulk petroleum, and principal items (ships, aircraft, missiles, tanks, vehicles, etc.).

e. These instructions also provide a system for scientific inventory management, establishing formulas and guidance for control of inventory decisions. Incorporation of statistical and economic concepts in the management of these inventories is required by the item manager. The EIP combines a statistical representation of demand, safety level quantities, and minimum operating level quantities into a system which emphasizes a balance between ordering costs and holding costs.

f. Exceptions to the EIP are:

(1) Compelling military reasons.

(2) Spoilage or loss.

(3) Deterioration.

(4) Seasonal buys.

(5) Economic production runs.

(6) Unavailable storage space.

(7) Unit pack or industry selling practices which would dictate a different quantity.

(8) Procurement funds not available.

(9) Shelf-life limits.

2. Economic Order Concept. The economic order concept is a method of comparing the elements of the cost to supply an item. Order quantities and safety levels are determined so as to minimize total variable costs of stockage as outlined in DODI 4140.39. The Total Variable Cost (TVC) is the sum of holding costs, ordering costs, and shortage costs. Comprising these costs are such items as interest, obsolescence, wages, storage space costs, taxes, insurance, paperwork, telephone, etc. The EOQ model allows the item manager to find the particular quantity to order which minimizes these total inventory costs. To differentiate between variable and fixed costs, consider that costs are "fixed" if they remain constant should 50 percent of the workload be eliminated.

3. Economic Order Quantity.

a. The purpose of the EOQ model is to find that quantity of an item to order which minimizes total inventory costs. In the current EOQ model, as developed by the Inventory Research Office (IRO), the three component costs involved in the computation are the ordering, holding, and shortage costs. A brief explanation of the cost elements should help you to better understand the model.

b. Ordering or procurement costs are basically those costs incurred in getting an item into the inventory. Each time an order is placed for an item, these costs are realized. They are then expressed as a dollar cost per order. Ordering costs begin with the initiation of the Procurement Work Directive (PWD), continue through its processing, and finally end with the payment of the supplier. Wages constitute a major portion of the cost. All functional elements considered in the cost to order are outlined in AR 710-1, Centralized Inventory Management of the Army Supply System.

c. An estimate is made of the variable costs for all the ordering operations that would be involved for each additional replenishment order. This cost estimate must consider wages, materials, and equipment required. As wages change or if the PWD process changes significantly, then an update should take place. This information will be recorded in the Materiel Management Decision (MMD) file. It will appear in sector 01, segment 02 as Cost to Procure, either stocked or nonstocked.

d. Holding costs are basically those costs incurred in order to maintain inventories. These costs are used by managers in much the same way as they are used in industry. Holding costs are a good indicator of efficiency in the management of inventories. They can be related to storage space, inventory turnover rates, obsolescence, total demands, etc. In industry, the holding costs bear a direct relationship to the profits, while in the Army the emphasis is on cost as a management tool. That is, they are used to achieve the management goal of optimum effectiveness in support of the troops in the field (readiness).

e. The holding costs consist of such elements as:

(1) Investment or discount cost. Government investment in inventory is paid for by withdrawing money from the private sector where it could be earning interest.

(2) Obsolescence. This is usually caused by either technological advances or overforecasting of requirements. It constitutes one of the largest elements of the holding cost.

(3) Storage cost. The storage cost includes the amortized cost of the storage facilities and the care of the materiel in storage.

(4) Other losses. Included in this category are such things as pilferage, shrinkage, and inventory adjustments.

f. These holding costs are normally expressed as a percentage of average inventory value; i.e., 20 percent per year to hold. Using inventory holding costs as a percentage of the product is convenient because regardless of the price of the product, the percentage can be applied. Like the ordering cost, the holding cost is found in the MMD file in sector 01, segment 02, expressed as a percentage of unit price.

g. Shortage costs are the most difficult of the cost parameters to define. These costs are a function of other management decisions.

(1) The average number of days to be forecast for delay in availability of materiel.

(2) The funds available for inventory levels.

h. These elements are subject to a great deal of variation in the system. Therefore, the system must have the capability to regulate inventory levels in conjunction with these two elements. The management decisions affecting the implied shortage costs to be used in supply operations and budget preparation are not within the realm of this block. Therefore, the calculation of a shortage cost will not be considered in the manual calculation of the EOQ.

i. The shortage parameter lambda (λ) may be determined through the manipulation of these elements. For example, a desired level of performance may be specified in terms of the average number of days forecast for delay in materiel availability. This, in turn, fixes the shortage parameter and affects the resultant funds required for the inventory. On the other hand, if the funding level is set, this fixes the shortage parameter and affects the level of performance that may be obtained.

4. Wilson EOQ Model.

a. In order to provide you with the concept of how an inventory model works, the Wilson EOQ Model will now be discussed. The Wilson EOQ Model seeks to find that particular quantity to order which minimizes total inventory costs.

b. This model considers two variables--ordering costs and holding costs--to determine the order quantity. The Wilson EOQ approximation is lower bound than the true optimal EOQ, but its simplicity of calculation and close approximation to the optimum have proved worthwhile.

c. The combination of ordering costs and holding costs produces the total variable cost. A change in either of these costs will affect the other and, consequently, the total variable cost. For instance, ordering costs will vary with the number of orders placed and may be reduced for any given item by placing fewer, but larger orders. The result is an increase in average monthly inventory with a corresponding increase is holding costs. Conversely, holding costs may be reduced by placing more frequent but smaller orders. This decreases holding costs while increasing ordering cost. How for a look at how this works.

d. There is a level at which the combined variable costs of ordering and holding inventory are at a minimum. The size of the order which produces this result is known as the EOQ.

(1) For example, let's examine an item having yearly demands amounting to $1,200. The following table illustrates how total variable costs are affected by the size and frequency of orders. In this example, the cost of ordering amounts to $4 per order and the cost of holding equals 10 percent of the dollar value of the average inventory. The average inventory is found by taking the value of each order and dividing this by 2.

VALUE OF ORDER	NO. ORDERS PER YEAR	VARIABLE ORDERING COSTS	AVERAGE ACTIVE INVENTORY	VARIABLE HOLDING COSTS	TOTAL VARIABLE COSTS
$1,200	1	$4	$600	$60	$64
600	2	8	300	30	38
400	3	12	200	20	32
300	4	16	150	15	31*
240	5	20	120	12	32
200	6	24	100	10	34

*Least total variable cost

(2) First, what would happen if we placed an order for $1,200? Since this is a year's supply, we would order the item only once during the year and the total variable cost of ordering would be $4. The value of the stock on hand would decrease from $1,200 at a rate of $100 per month until exhausted. At the end of the year, the average value of the stock on hand would be $600 (or one-half the value of the original order). The variable holding costs would amount to 10 percent, or $60. The sum of the two variable costs would be $4 plus $60, or a total of $64.

(3) Suppose, however, we order the item twice a year, making the value of each order $600. This would double the ordering costs and cut the holding costs in half. Total variable cost would be $38 instead of $64. However, by placing four orders for $300 each, the total variable cost would be the least of all. Beyond this point, the increase in variable ordering costs would always be greater than the decrease in variable holding costs, as can be proved by extending the table to include more and more orders per year. Therefore, our EOQ would be 300 and we would place four orders (once every 3 months).

e. As depicted in the preceding table, a reduction in one set of costs causes an increase in the other set of costs. In any system of variables, there are numerous combinations which result in advantages for one element at the cost of effectiveness or efficiency in other elements. The application of EOQ principles to a given item or a group of items will cause the sum of the two sets of costs to be lower than under any other system of replenishment. This can result from balancing or equating the two sets of costs (or by reducing one set of costs without proportionally increasing the other set of costs). Under many replenishment systems where demands are relatively constant, items are ordered at regular intervals. Such systems result in maintaining, on the average, a constant level of supply for all items in the group. Under other systems, the number of orders may vary according to demand. For items with a high-dollar demand value, orders may be placed more frequently than for low-dollar demand items, in which case fewer and larger orders, relative to demand, are placed. In addition, under some of these systems consideration is properly given to special characteristics such as limited shelf life, space requirements, and seasonal demand.

f. It is Army policy that operating levels and replenishment cycles for each secondary item will be adjusted to that point where total variable costs are minimized. This condition can generally be obtained by application of the economic order principle which attempts to equate the cost to order to the cost to hold. Where cost to order equals cost to hold, the total variable costs will be minimized (see Figure 1).

g. Now for a look at the formula used to solve the Wilson EOQ Model. While the IRO-EOQ Model currently in use by AMC is more complex, this formula illustrates the theory of balancing holding costs against ordering cost to obtain the minimum variable cost.

h. This formula is used to determine the EOQ or the quantity applicable to the procurement (EOQ) cycle. This can be converted to time (months) by dividing "Q" by the forecasted average monthly demand (FAMD) to give us the length of the procurement (EOQ) cycle. Currently, DOD 4140.1-R states that the EOQ shall be limited to a maximum of 24 months and, at the wholesale level, a minimum of the lesser of the administrative leadtime or 6 months. In some cases, we are given a specific order cost for a particular item:

$$Q = \sqrt{\frac{2CY}{HU}}$$

Where Q = The EOQ
Y = The yearly demands
C = The cost to place an order
H = The cost to hold an item--expressed as a percentage of the dollar value of the average inventory
U = Unit cost

$Y = 1{,}200$ $\qquad Q = \sqrt{\frac{2(C)(Y)}{HU}}$

$U = 1.00$ $\qquad Q = \sqrt{\frac{2(4)(1{,}200)}{(.10)(1.00)}}$

$H = 10\%$ $\qquad Q = \sqrt{\frac{9{,}600}{.1}}$

$C = \$4$ $\qquad Q = \sqrt{96{,}000}$

$$Q = 309.8386 \approx 310$$

NOTE: These are the same costs used in paragraph 4d. The EOQ varies slightly because we are not restricting ourselves to whole month increments in these examples as we did before. In addition, when working both EOQ and safety level problems, we always round up for the sake of readiness (stock availability).

5. Procurement (EOQ) Cycle Time. The EOQ model has answered one of the basic questions of "how much to buy" as shown in the example. In order to answer the second question, "how often to buy," this quantity must be converted into time. This is referred to as procurement cycle or EOQ cycle time. In the Commodity Command Standard System (CCSS), the procurement (EOQ) cycle time (how often to buy) is calculated in months and is converted to procurement (EOQ) cycle requirement (EOQ)[1] by using forecasted demands. This cycle time can be determined by dividing the EOQ by the net monthly demands. This will give the EOQ expressed in months:

$$\text{Procurement (EOQ) cycle time} = \frac{\text{EOQ}}{\text{NMD}}$$

Where EOQ = Economic order quantity
NMD = Net monthly demands

Example: NMD = 100 EOQ = 310

$$\text{Procurement (EOQ) cycle time} = \frac{310}{100} = 3.1 \text{ months}$$

6. Safety Levels.

a. Related to the EOQ is the desired protection against stock depletion referred to as a safety level. The ideal situation is often thought of by the inventory manager as always avoiding both "zero balances" and "backorder" positions. The existence of shortages suggests that some additional stock should be placed on the shelf to protect against uncertainty of demand during the resupply time. Therefore, action must be taken to adjust for unexpected demands from customers or delays in delivery from acquisition since shortages can be quite costly in terms of unit readiness. The safety level is an attempt to overcome this problem. However, 100% protection is theoretically impossible.

b. Recall from the IRO-EOQ model that the total variable cost is composed of inventory holding cost, procurement ordering cost, and a shortage (backorders) cost. The cost applied to the backorders is adjusted by the shortage cost parameter lambda (λ). Now, the larger the lambda (λ) value, the most costs are attributed to requisitions short and more safety level is required to minimize total costs. The more safety level required equals fewer backorders on one hand, but more money is required for stock on the other hand.

c. Variable Safety Level (VSL). VSL is based on probability principles, analysis of historical data, and monetary resources available. The safety stock may either be set at some point to provide an expected level of availability (e.g., a policy that states 95 percent of all demands will be met directly from stock on hand), or the safety stock may be set at a point that minimizes the cost of shortages and the costs of carrying additional inventory. The safety stock is carried in addition to stocks on hand to meet normal demands in order to achieve the target stock availability.

d. Stock availability, as used within AMC, refers to the number of units of an item demanded which can be supplied from stock currently on hand. A 95 percent availability level means that if 1,000 units of

[1] The EOQ may also be referred to as the procurement cycle requirement or the reorder cycle requirement.

an item are demanded during the year, 950 of the units can be supplied immediately. The remaining 50 will be short and backorders generated. The computation of a VSL considers such data as the frequency of demands, the size of the average order, the implied requisition shortage cost, and reorder frequencies.

e. The models used in CCSS to calculate VSL are based on the procurement (acquisition) leadtime demand and are too involved to cover at this time. Therefore, a less complicated example of a safety level computation will be used to provide a basic understanding of the VSL. During the past year, a particular item has averaged a demand of 180 units. A 95 percent availability rate for the item has been specified. Figure 2 depicts this graphically. The shaded area on the right side represents 5 percent; in this case, the percentage of time requisitions cannot be satisfied from stock on hand (backorder/due-out). The standard deviation (σ)[2] is given as 30 units. Based on this information, the quantity of stock required to obtain a 95 percent level of availability can be determined. The manager must reorder at some point (R). Therefore, his safety level will be R - 180. Now to find R, the normal distribution table (Figure 1) must be referred to. Finding the value of .9505 in the table, since is the closest to the desired level of availability of 95 percent, a K value of 1.65 is found. This represents the number of standard deviations to the right of the mean that must be assumed to obtain the 95 percent level. Since one standard deviation is 30 units, then 1.65 standard deviation is (1.65 x 30 = 49.5) approximately 50 units. Therefore, if reorders are made when the stock level falls to 230 or (180 + 50) units, the inventory manager will be able to fill his requirements 95 percent of the time.

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}}$$

where σ = standard deviation

x = actual yearly demands

μ = mean (average yearly demands)

n = total number of years used

COSTS OF VARIOUS AVAILABILITY LEVEL POLICIES

SERVICE LEVEL DESIRED, %	NO. OF STD DEV (σ)	SAFETY STOCK RQD UNITS	COST/YR $	COST OF 1% INCREASE IN AVAILABILITY
50	0	0	0	$ 4.33 = (130-0)/30
80	.85	26	130	6.50 = (195-130)/10
90	1.29	39	195	11.00 = (250-195)/5
95	1.65	50	250	25.00 = (350-250)/4
99	2.33	70	350	127.78 = (465-350)/.9

Mean leadtime demand (μ) = 180
Standard deviation of leadtime demand (σ) = 30
Cost per year to carry 1 unit of safety stock = $5

[2] Briefly, the standard deviation is arrived at by subtracting the mean (180) from the actual demands for each yearly period available and then squaring the result. Each squared value is then added together and divided by the total number of years of actual demand data used (n). Then, take the square root of this quotient to obtain the standard deviation. DOD 4140.1-R states that no item will be given a safety level of more than two standard deviations of leadtime demand or more than the leadtime demand, whichever is less.

f. It should be obvious that there is a relationship between the cost of carrying safety stock and increased availability levels. Fund constraints will often drive this availability rate, since the higher the desired level of availability will show this relationship. The safety stock required to provide several different levels of stock availability have been calculated along with the associated costs of carrying these different amounts of safety stock.

g. The right-hand column of the table indicates the approximate cost for a 1 percent increase in the level of availability. The cost increases quite rapidly relative to the stock availability level required. This is shown graphically in Figure 3. It becomes clear that once the availability level passes about 95 percent, few items would be stocked at this level unless they are deemed very essential to overall readiness.

h. This example should provide some insight as to how the actual model works in calculating a VSL. It should be pointed out that any set of decision rules developed from the IRO-EOQ models will be sensitive to extremes of cost and demand.

7. EOQ/VSL Simulation. The EOQ/VSL simulation provides an opportunity for the item manager to vary the various elements affecting the EOQ and VSL. From this simulation, the manager can readily see the effects of element changes on the outcome and better understand the system.

8. Supply Performance Analyzer (SPA). The SPA produces estimates of the relationship between supply performance and a given funding level. SPA output are shortage cost parameters (lambdas) to be used in the EOQ/VSL computations. Essential items are given a higher stock availability target (90 percent) than non-essential items (85 percent). The SPA performs simulations similar to budget stratification. The larger the shortage cost, the larger the safety level on a given item which, in turn, improves supply performance and, thereby, increases funding requests.

9. Repair Safety Level. The amount of stock to be on hand to guard against a due out occurring due to fluctuation to repair lead-times and gross demands. This level will only appear on reparable items in the Medium/High Dollar Value path. There is no override capability for the item managers. It will not affect the Requirements Objective directly, but has replaced the use of "Procurement Safety Level" in the computation of the Repair Action Point (RAP). The Repair Safety Level will use the same LAMBDA value used in the Procurement Safety Level computation. This is due to the fact that the cost of a backorder is the same whether it is filled through repair or procurement. In addition to the LAMBDA value, the Repair Safety Level utilizes the following elements in its calculation: repair lead-time, gross demands, and overhaul price.

10. Summary.

a. There are few management responsibilities in the world today that compare with the scope and complexity of the management of DOD inventories. Manyproblems arise due to the size and diversity; others are due to the many echelons through which an extended supply pipeline must reach out to support forces all over the world.

b. Of primary concern are the size and value of the inventory. If it is larger than necessary to meet current and anticipated needs, then the cost of maintaining the system is significantly increased. As discussed, these costs come in many forms from obsolescence to loss of readiness. Careless management of inventories in one area affects the stock positions and management of other items by tying up critical monetary resources.

c. Unfortunately, the possibility exists for wrong decisions at the end of supply control studies. These bad decisions result in loss of time and money and are most often attributed to faulty input data. Hopefully, this discussion of EOQ/VSL will provide you, the item manager, with at least a brief understanding of how the system operates. To avoid backorders and overabundance is a difficult task at best; yet, it is of such importance that item managers must always operate at their best. A good understanding of how to collect, evaluate, and analyze demand data is a necessity. The item manager is the key to the success or failure of providing adequate support to the troops in the field, while at the same time holding the cost of support to the absolute minimum.

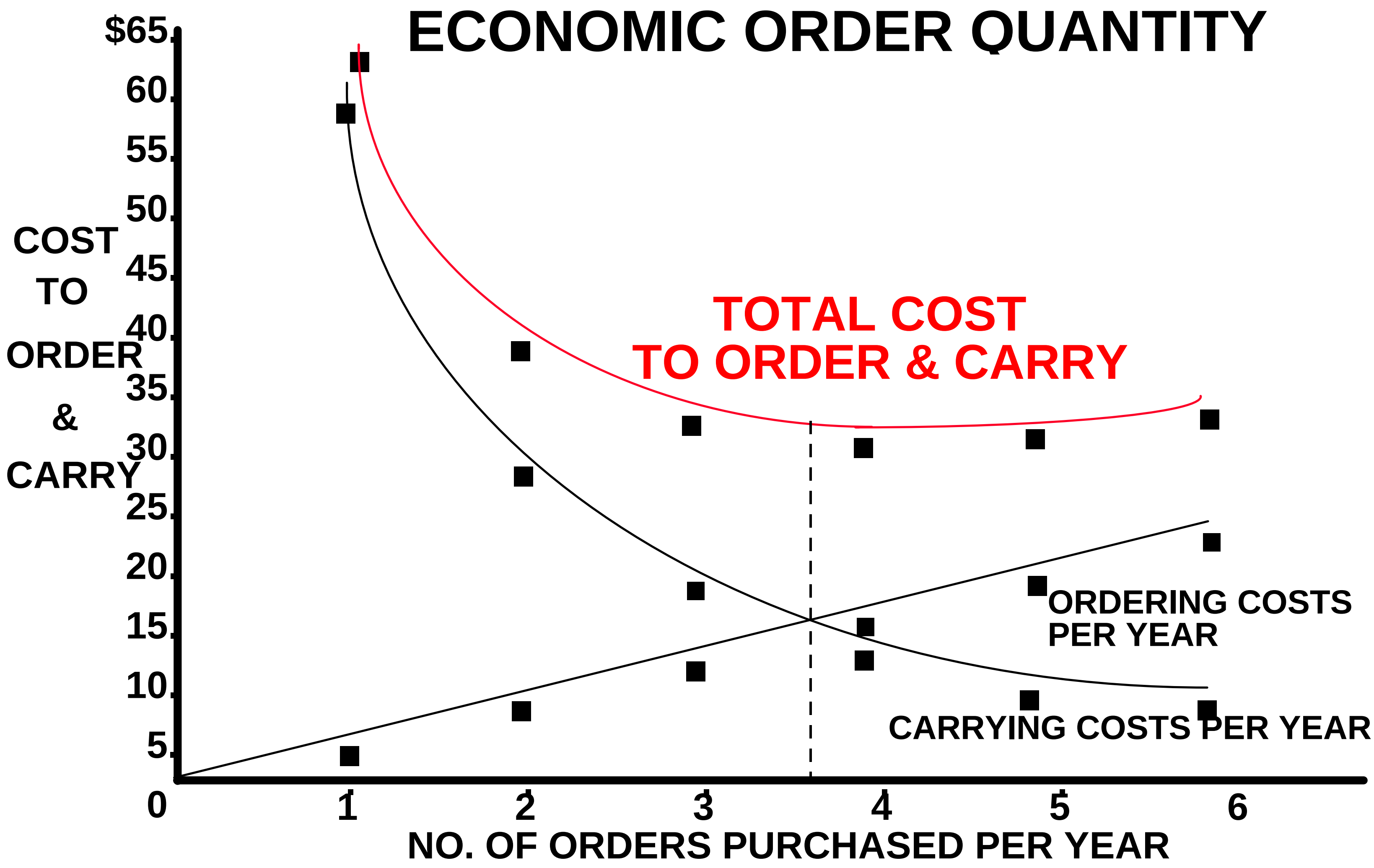
ECONOMIC ORDER QUANTITY
$65
60
55
50
45
40
35
30
25
20
15
10
5
0
COST
TO
ORDER
&
CARRY
TOTAL COST
TO ORDER & CARRY
ORDERING COSTS
PER YEAR
CARRYING COSTS PER YEAR
1
2
3
4
5
6
NO. OF ORDERS PURCHASED PER YEAR

Figure 1

DETERMINATION OF SAFETY STOCK

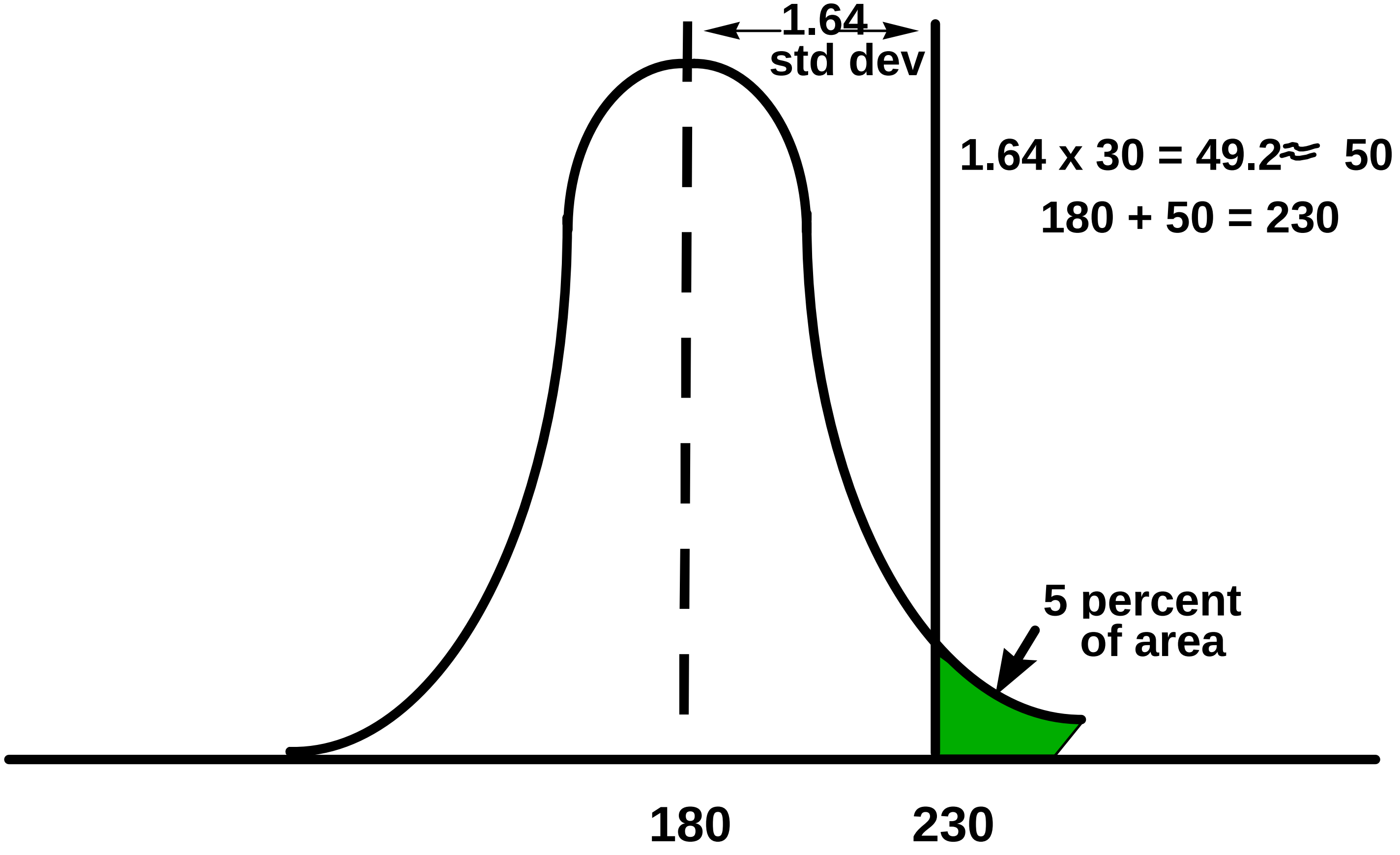

Figure 2

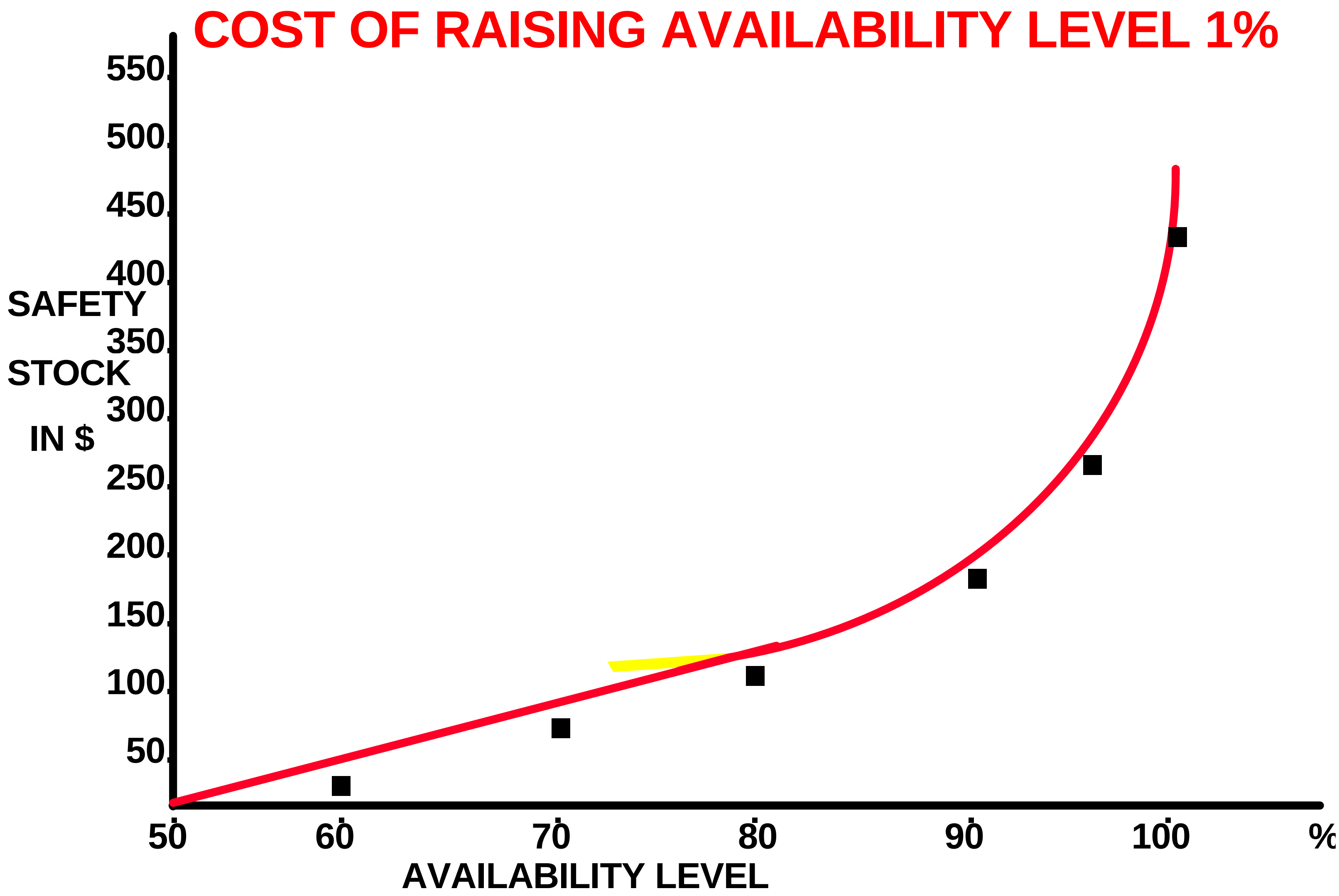
COST OF RAISING AVAILABILITY LEVEL 1%
550
500
450
400
SAFETY
350
STOCK
300
IN $
250
200
150
100
50
50
60
70
80
90
100
%
AVAILABILITY LEVEL

Figure 3

NORMAL AREA CURVE TABLE

k.	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
+0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
+0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
+0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.4443	0.6480	0.6517
+0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6737	0.6772	0.6808	0.6844	0.6879
+0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
+0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
+0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
+0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8079	0.8106	0.8133
+0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
+1.0	0.8643	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
+1.1	0.8849	0.8665	0.8668	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
+1.2	0.9032	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
+1.3	0.9192	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
+1.4	0.9332	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
+1.5	0.9452	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
+1.6	0.9554	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
+1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
+1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
+1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
+2..0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

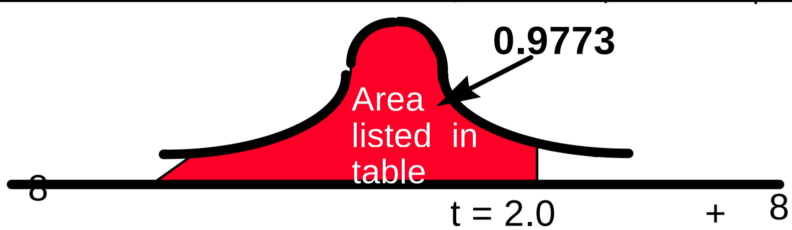

Figure 4

THIS PAGE INTENTIONALLY LEFT BLANK