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# ASSOCIATES LOGISTICS EXECUTIVE DEVELOPMENT CORRESPONDENCE COURSE (ALEDC) PHASE IV: DECISION SCIENCES

Welcome to ALEDC Phase IV. This course will provide a general introduction into Statistics and Operations Research. These subjects sometimes cause concern to students with relatively little math background. This course should provide some structure to quantifiable common sense that you have already reasoned with. You will soon realize that you use man of these fundamental concepts in your life already. Questions concerning course administration should be directed to the AIPD office, DSN 927-3335 or Comm (804) 878-3335. Questions concerning course content should be directed to ALMC, DSN 539-4254 or Comm (804) 765-4254.

The text you received was tailored exclusively for this course. Chapters were selected from various texts and compiled in such a way to present a logical progression through the introduction of Statistics and Operations Research. All topics are presented in the text except Multi-attribute Decision Analysis. The last few sections of this supplement provide instruction, a practical exercise, and a solution in this topic area. All other instruction is to come directly from your text.

Please note the section of this section titled “Lesson Assignments”. This section assigns suggested problems from each chapter covered in the text. The sections following provide detailed solutions to these assigned problems. I recommend that you attempt the problems without using the solution first. Use the solution set to verify your work. These problems will not be graded. They are for your use only.

There will be one exam for the course. You will be allowed up to five hours to complete the exam. The exam will be open book, open note. The problems will be representative of the assigned homework and practical exercises. If you can do the assigned work, you can expect to do well on the exam. If you cannot do the assignments, do not take the exam. Contact the course director.

You are required to designate an individual to act as Test Control Officer (TCO) to administer your examination. This person must be a certified TCO who is familiar with examination procedures. Complete Fort Lee Form 432-FL and return to:

> The Army Institute for Professional Development
> ATTN: ATIC-IPS (Student Services)
> Newport News, VA 23628-0001
> DSN: 927-3335/5442 or Comm (804) 878-3335/5442

Upon receipt of this request, the correspondence office will forward an examination to your TCO for administration. This form is included in the front of this book.

Have fun and good luck with the course.

## NOTICE TO STUDENTS

In order to provide quality training, ALMC has the policy of tailoring correspondence courses to mirror classroom instruction as closely as possible. This correspondence course has been compiled using materials that are used in the resident mode of instruction. Therefore, you will find the ALMC numbering system used throughout this text. However, as AIPD now administers this course, the course number indicated on the cover of this text as well as the exam identification number is consistent with format used by AIPD.

## Relationship to the Terminal Learning Objectives

The interaction between the decision maker and his staff is a critical skill to insure success in today's "resource constrained" Army. Upon completion of this phase of ALEDC, the student will gain an appreciation of some basic Decision Sciences techniques and capabilities. Development of these techniques will enhance the critical decision process governing the management of constrained resources.

## Enabling Learning Objectives

**Probability and Statistics**

Given a problem scenario, recognize if a data set represents the population or a sample.

If a data set is a population, determine the population mean, population variance and the population standard deviation.

Define and/or review the following probability terms:

a. Mutually Exclusive Events and Collectively Exhaustive Events
b. Partitioning of the Sample Space
c. The Law of Total Probability
d. A Priori Probabilities
e. Joint Probabilities and Marginal Probabilities
f. Bayes' Theorem

Compute and interpret probabilities using:

a. The Law of Total Probability
b. Bayes' Theorem
c. Probability Trees
d. Probability Tables or Contingency Tables

Define the Normal Curve and discuss its characteristics.

Provide practical applications for using the Normal Curve.

Define, Compute and Interpret a Z-Score.

Transform a Z-score to an X-score.

Learn how to read a Normal Curve Table with Cumulative Probabilities.

Use the Z-Score to find Cumulative Probabilities in the Cumulative Z-Table.

Find areas of probability under the normal curve bounded by given X-scores:

a. Find $P(X < x)$
b. Find $P(X > x)$
c. Find $P(X_1 < X < X_2)$

Find the values of X that border a percentage of the curve.

For example, determine the following:

a. Find the value of X below which 45% of the normal curve falls.
b. Find the value of X above which 27% of the normal curve falls.
c. Find the two values of X that border the middle 60% of the normal curve.

If a data set represents a sample:

a. Determine the mean, the median, and the mode.
b. Determine the range, the variance, the standard deviation, and the interquartile range.
c. Construct a $(1-\alpha)$ 100% Confidence Interval for the population mean.

If a data set represents a sample, conduct a hypothesis test about the population mean. The hypothesis test should include the following:

a. Develop the null and alternative hypotheses.
b. Develop the Decision Rule.
c. Calculate the appropriate Test Statistic.
d. Infer the appropriate conclusion.

**Management Science**

Setup a payoff table for a decision analysis problem, including the alternatives, states of nature, and consequences.

Apply the following decision analysis techniques under conditions without knowledge of probabilities:

a. Optimistic approach
b. Conservative approach
c. Minimax regret approach

Determine and interpret the expected value of alternatives under conditions with knowledge of probabilities.

Setup a decision matrix for evaluating alternatives with multiple attributes.

Apply dominance and satisficing techniques to screen alternatives in a multi-attribute decision analysis.

Apply appropriate scaling methods for qualitative and quantitative data in a multi-attribute decision analysis.

Use attribute weights to determine consolidated scores and compare alternatives in a multi-attribute decision analysis.

Define the following linear programming terms:

a. Decision variable
b. Objective function
c. Constraint
d. Feasible region
e. Optimal solution

Identify the application of linear programming techniques in a problem solution and formulate a linear programming model.

Graphically solve a linear programming model and identify the feasible region, optimal solution, and binding constraints.

Represent a transportation problem in a network representation and formulate the corresponding linear programming model.

Represent an assignment problem in a network representation and formulate the corresponding linear programming model.

Represent a transshipment problem in a network representation and formulate the corresponding linear programming model.

Define the following project scheduling terms:

f. Activity
g. Immediate predecessor
h. Project network
i. Earliest start/earliest finish
j. Project completion time
k. Latest start/latest finish
l. Slack
m. Critical path

Given a set of activities, activity times, and immediate predecessors, construct a project network.

Determine the estimated project completion time, activity slack times, and critical path of a project network.

Define the measures of holding cost, ordering cost, and backorder cost in an inventory situation.

Determine the optimal order quantity, reorder point, and total annual inventory costs for an economic order quantity (EOQ) model.

Determine the optimal order quantity, planned backorders, and the total annual inventory costs for an inventory model with planned shortages.

## ALEDC Phase IV Correspondence Textbook Policy

As a student in the correspondence version of the ALEDC Phase IV, you will receive a textbook as part of your course materials. The Decision Sciences for Logisticians text represents a considerable investment to the Army and is intended to be used many times.

AIPD has experienced a significant number of students that keep the texts for an excessive period of time. This creates out-of-stock conditions at our distribution point and causes other students to wait for these texts to be returned to AIPD.

To help reduce this problem in ALEDC Phase IV, the following policy will be in affect regarding the textbook. Students will receive the text to complete the correspondence course. Upon completion of the final examination, the student's examination will be held and no credit will be recorded until the text is returned to AIPD. In addition, no subsequent materials will be forwarded to the student until the text has been returned. No exception to this policy will be made for any reason to include pending promotion boards, retirement year ending dates, or course completion deadlines. Thank you for your help in this matter.

# LESSON ASSIGNMENTS

| CHAPTER NUMBER | CHAPTER NAME | READING ASSIGNMENT | PROBLEM ASSIGNMENT |
|---|---|---|---|
| Chapter 1 | Data and Statistics | pp. 3-17 | None |
| Chapter 2 | Descriptive Statistics II: Numerical Methods | pp. 25-38 | pp. 31-32 # 3, 4, 7, 8<br>pp. 38- 40 # 13, 14, 18 b, c; 21 a, b, c |
| Chapter 3 | Introduction to Probability | pp. 85-86, 90-118 | pp. 94-95 # 7, 10; pp. 98-99 # 18, 21<br>pp. 104-105 # 25, 28; pp. 111 # 32, 33<br>pp. 118-119, # 41, 42, 43 |
| Chapter 4 | Continuous Probability Distributions | pp. 135-146 | pp. 146-147 # 11, 13, 18, 19 |
| Chapter 5 | Sampling and Sampling Distributions | pp. 165-166, 171-187 | pp. 174-175 # 11, 15<br>pp. 187 # 18 a, b |
| Chapter 6 | Interval Estimation | pp. 205-218 | pp. 212-213 # 1, 2, 6, 11<br>pp. 219-220 # 15, 16, 19, 22 |
| Chapter 7 | Hypothesis Testing | pp. 241-266 | pp. 254-255 # 9, 10 a, b, d, 11, 15 a, c<br>pp. 261-263 # 19, 21, 25 a, b, c<br>pp. 266-267 # 29, 30, 31, 33 a, b, c, 34 a |
| Chapter 8 | Intro to Problem Solving and Decision Making | pp. 283-293, 297-300 | None |
| Chapter 9 | Decision Analysis | pp. 313-321 | pp. 344-345 # 2, 4 a, b, c, d, f<br>pp. 350-351 # 15 a, d, e |
| Chapter 11 | Introduction to Linear Programming | pp. 383-417 | pp. 419 # 14, pp. 421 # 22<br>pp. 426 # 38 (formulate – do not solve)<br>pp. 426 # 39 |
| Chapter 13 | Transportation, Assignment and Transshipment Problems | pp. 491-510 | pp. 515-516 # 4 a, b (do not solve), 5 a, b<br>pp. 520 # 12 a, b (do not solve)<br>pp. 525 # 26 a, b |
| Chapter 14 | Project Scheduling: PERT/CPM | pp. 537-553 | pp. 560-563 # 3, 4, 8 a, b, d, 11, 12 |
| Chapter 15 | Inventory Management: Independent Demand | pp. 573-582, 586-590 | pp. 608 # 4, 5<br>pp. 610 # 15, 17 (only answer 1$^{st}$ question) |
| Chapter in this guide | Multi-attribute Decision Analysis | Multi-attribute Decision Analysis chapter | Work Multi-attribute Decision Analysis practical exercise |

# CHAPTER 2 (DESCRIPTIVE STATISTICS) SOLUTIONS

**Page 31 # 3)**

(a) 20$^{th}$ percentile

**Step 1**

15
**20**
25
25
27
28
30
34

**Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right)n = \left(\frac{20}{100}\right)8 = 1.6}$$

**Step 3:** Round 1.6 up to 2.
2$^{nd}$ value is **20**, the 20$^{th}$ percentile.

(b) 25$^{th}$ percentile

**Step 1**

15
**20**
**25**
25
27
28
30
34

**Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right)n = \left(\frac{25}{100}\right)8 = 2}$$

**Step 3:** 2$^{nd}$ value is 20, 3$^{rd}$ value is 25.
25$^{th}$ percentile is (20+25)/2 = 45/2 = **22.5**.

(c) 65$^{th}$ percentile

**Step 1**

15
20
25
25
27
**28**
30
34

**Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right)n = \left(\frac{65}{100}\right)8 = 5.2}$$

**Step 3:** Round 5.2 up to 6.
6$^{th}$ value is **28**, the 65$^{th}$ percentile.

(d) 75$^{th}$ percentile

**Step 1**

15
20
25
25
27
**28**
**30**
34

**Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right)n = \left(\frac{75}{100}\right)8 = 6}$$

**Step 3:** 6$^{th}$ value is 28, 7$^{th}$ value is 30.
75$^{th}$ percentile is (28+30)/2 = 58/2 = **29**.

**Page 31 # 4)**

| | |
|---|---|
| **53** | |
| **53** | **Mode = 53 (occurs 3 times)** |
| **53** | |
| 55 | |
| 57 | |
| **57** | **Median = 57 (middle value)** |
| 58 | |
| 64 | |
| 68 | |
| 69 | **Mean:** $\bar{X} = \frac{\sum X_i}{n} = \frac{657}{11} = 59.73$ |
| 70 | |
| 657 | |

**Page 32 # 7)**

(a) $$\bar{X} = \frac{\sum X_i}{n} = \frac{1380}{30} = 46\,\text{min.}$$

(b) Yes. Not far from 45 minutes.

(c) $$\text{Median} = \frac{45.0 + 52.9}{2} = \frac{97.9}{2} = 48.95\,\text{min.}$$

(d) $25^{th}$ percentile

**Step 2:** $$i = \left(\frac{p}{100}\right)n = \left(\frac{25}{100}\right)30 = 7.5$$

**Step 3:** Round 7.5 up to 8. $8^{th}$ value is 7.0, the $25^{th}$ percentile.

$75^{th}$ percentile

**Step 2:** $$i = \left(\frac{p}{100}\right)n = \left(\frac{75}{100}\right)30 = 22.5$$

**Step 3:** Round 22.5 up to 23. $23^{th}$ value is 70.4, the $75^{th}$ percentile.

(e) $40^{th}$ percentile

**Step 2:** $$i = \left(\frac{p}{100}\right)n = \left(\frac{40}{100}\right)30 = 12$$

**Step 3:** $12^{th}$ observation is 28.8, $13^{th}$ observation is 29.1, $40^{th}$ percentile = (28.8+29.1)/2 = 28.95 minutes
40% of people listen to recorded music 28.95 minutes or less a day.

**Page 32 # 8)**

(a) Mode = 29 years $$\bar{X} = \frac{\sum X_i}{n} = \frac{775}{20} = 38.75\text{ years}$$

(b) Median = (37+40)/2 = 77/2 = 38.5 years (older than 35.1 years median age of all adults)

**Page 32 # 8 continued)**

(c) $25^{th}$ percentile

**Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right) n = \left(\frac{25}{100}\right) 20 = 5}$$

**Step 3:** $5^{th}$ observation is 29, $6^{th}$ observation is 30
$25^{th}$ percentile = (29+30)/2 = **29.5**

$75^{th}$ percentile

**Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right) n = \left(\frac{75}{100}\right) 20 = 15}$$

**Step 3:** $15^{th}$ observation is 46, $16^{th}$ observation is 49
$75^{th}$ percentile = (46+49)/2 **= 47.5**

(d) $32^{nd}$ percentile

**Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right) n = \left(\frac{32}{100}\right) 20 = 6.4}$$

**Step 3:** Round 6.4 up to 7. $7^{th}$ value is 31, the $32^{nd}$ percentile.
32% of the individuals who work at home are 31 years of age or younger.

**Page 38 # 13)**

10
12
16
17
20 **Range = 20 – 10 = 10**

$25^{th}$ percentile: **Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right) n = \left(\frac{25}{100}\right) 5 = 1.25}$$

**Step 3:** Round 1.25 up to 2. $2^{nd}$ value is 12, the $25^{th}$ percentile.

$75^{th}$ percentile: **Step 2:** $$\mathbf{i = \left(\frac{p}{100}\right) n = \left(\frac{75}{100}\right) 5 = 3.75}$$

**Step 3:** Round 3.75 up to 4. $4^{th}$ value is 17, the $75^{th}$ percentile.
**IQR = $Q_3 - Q_1$ = 17 – 12 = 5**

**Page 38 # 14)**

| X | $X_i$-sample mean | $(X_i\text{-sample mean})^2$ |
|---|---|---|
| 10 | 10-15 = -5 | -5 * -5 = 25 |
| 12 | 12-15 = -3 | -3 * -3 = 9 |
| 16 | 16-15 = 1 | 1 * 1 = 1 |
| 17 | 17-15 = 2 | 2 * 2 = 4 |
| 20 | 20-15 = 5 | 5 * 5 = 25 |
| 75 | | 64 |

$$\mathbf{\bar{X} = \frac{\sum X_i}{n} = \frac{75}{5} = 15}$$

$$\mathbf{s^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{64}{4} = 16}$$

$$\mathbf{s = \sqrt{16} = 4}$$

**Page 39 # 18)** (b)

| **X** | **$X_i$-sample mean** | **($X_i$-sample mean)$^2$** |
|---|---|---|
| 28 | 28-48.33 = -20.33 | -20.33 * -20.33 = 413.3089 |
| 42 | 42-48.33 = - 6.33 | - 6.33 * - 6.33 = 40.0689 |
| 45 | 45-48.33 = - 3.33 | - 3.33 * - 3.33 = 11.0889 |
| 48 | 48-48.33 = - 0.33 | - 0.33 * - 0.33 = 0.1089 |
| 49 | 49-48.33 = 0.67 | 0.67 * 0.67 = 0.4489 |
| 50 | 50-48.33 = 1.67 | 1.67 * 1.67 = 2.7889 |
| 55 | 55-48.33 = 6.67 | 6.67 * 6.67 = 44.4889 |
| 58 | 58-48.33 = 9.67 | 9.67 * 9.67 = 93.5089 |
| 60 | 60-48.33 = 11.67 | 11.67 * 11.67 = 136.1889 |
| **435** | | **742.0001** |

$$\bar{X} = \frac{\sum X_i}{n} = \frac{435}{9} = 48.33$$

$$s^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{742.0001}{8} = 92.75$$

$$s = \sqrt{92.75} = 9.63$$

(c) Very similar to the air quality index for Pomona. Anaheim is a little more variable.

**Page 40 # 21)**

| X | $X_i$-sample mean | ($X_i$-sample mean)$^2$ |
|---|---|---|
| 168 | 168-178 = -10 | -10 * -10 = 100 |
| 170 | 170-178 = - 8 | - 8 * - 8 = 64 |
| 174 | 174-178 = - 4 | - 4 * - 4 = 16 |
| 182 | 182-178 = 4 | 4 * 4 = 16 |
| 184 | 184-178 = 6 | 6 * 6 = 36 |
| 190 | 190-178 = 12 | 12 * 12 = 144 |
| **1068** | | **376** |

**(a) Range = 190 – 168 = 22**

**(b)** $$s^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{376}{5} = 75.2$$

**(c)** $$s = \sqrt{75.2} = 8.67$$

# CHAPTER 3 (INTRO TO PROBABILITY) SOLUTIONS

**Page 94 # 7)** No. Probabilities do not sum to 1.0.

**Page 95 # 10)** (a) $\frac{21{,}733}{51{,}745} = .42$ (b) $\frac{7{,}286}{14{,}012} = .52$

(c) $\frac{51{,}745}{(51{,}745 + 14{,}012 + 17{,}229)} = .624$ (d) $\frac{(21{,}733 + 7{,}286 + 10{,}682)}{(51{,}745 + 14{,}012 + 17{,}229)} = .478$

**Page 98 # 18)** (a) P(0) = .05 (b) P(4,5) = .10+.10 = .20 (c) P(0,1,2) = .05+.15+.35 = .55

**Page 99 # 21)**

(a) P(A) = $\frac{(20+12+6+3+1)}{(8+20+12+6+3+1)} = .84$ (b) P(B) = $\frac{(6+3+1)}{(8+20+12+6+3+1)} = .20$

(c) P(2 activities) = $\frac{12}{(8+20+12+6+3+1)} = .24$

**Page 104 # 25)**

(a) P(M) = $\frac{155}{254} = .61$ P(C) = $\frac{152}{254} = .598$ P(M $\cap$ C) = $\frac{110}{254} = .433$

(b) P(at least one perk) = P(M) + P(C) – P(M $\cap$ C) = .61 + .598 - .433 = .775

(c) P(no perks) = 1 – P(at least one perk) = 1 - .775 = .225

**Page 105 # 28)**

(a) P(Bus or Per) = P(Bus) + P(Per) – P(Bus $\cap$ Per) = .458 + .54 - .30 = .698

(b) P(no rental) = 1 – P(Bus or Per) = 1 - .698 = .302

**Page 111 # 32)**

(a)

| | **Marital Status** | | |
|---|---|---|---|
| **Age** | Single | Married | Totals |
| Under 30 | 77/140 = **0.55** | 14/140 = **0.10** | **0.65** |
| 30 or over | 28/140 = **0.20** | 21/140 = **0.15** | **0.35** |
| Totals | **0.75** | **0.25** | **1.00** |

(b) P(Under 30) = 0.65, P(30 or Over) = 0.35 (c) P(Single) = 0.75, P(Married) = 0.25

(d) P(Single ∩ Under 30) = 0.55 (e) P(Single | Under 30) = 0.55 / 0.65 = 0.85

(f) No, marital status is not independent of age, because P(Single) ≠ P(Single | Under 30)

**Page 111 # 33)**

(a)

| | Application Reason | | | |
|---|---|---|---|---|
| **Enrollment Status** | School Quality | School Cost or Convenience | Other | Totals |
| Full Time | 421/1929 = **0.218** | 393/1929 = **0.204** | 76/1929 = **0.039** | **0.461** |
| Part Time | 400/1929 = **0.207** | 593/1929 = **0.307** | 46/1929 = **0.024** | **0.539** |
| Totals | **0.426** | **0.511** | **0.063** | **1.00** |

Note: some of the totals may be slightly off due to rounding

(b) P(School Quality) = 0.426, P(Cost/Convenience) = 0.511, P(Other) = 0.063

(c) P(School Quality | Full Time) = 0.218/0.462 = 0.472

(d) P(School Quality | Part Time) = 0.207/0.539 = 0.384

(e) No, because P(School Quality) ≠ P(School Quality | Full Time)

**Page 118 # 41)** Let S = successful bid, F = failed bid, Y = additional info, N = no additional info

(a) P(S) = 0.50 (50-50 chance) (b) P(Y | S) = 0.75

(c) $$P(S\,|\,Y) = \frac{P(S)P(Y\,|\,S)}{P(S)P(Y\,|\,S) + P(F)P(Y\,|\,F)} = \frac{(0.50)(0.75)}{(0.50)(0.75) + (0.50)(0.40)} = \frac{.375}{.575} = .652$$

**Page 118 # 42)** Let DF = default, NDF = no default, M = miss payment, NM = did not miss payment

(a) P(DF) = 0.05, P(NDF) = 0.95 Also, P(M | NDF) = 0.20 and P(M | DF) = 1.00

$$P(DF\,|\,M) = \frac{P(DF)P(M\,|\,DF)}{P(DF)P(M\,|\,DF) + P(NDF)P(M\,|\,NDF)} = \frac{(0.05)(1.00)}{(0.50)(1.00) + (0.95)(0.20)} = \frac{0.05}{0.24} = .208$$

(b) Yes, since missing a payment implies a probability of default of .208, which is greater than .20.

**Page 119 # 43)** Let A = accident, N = no accident, M = men, W = women

P(A | M) = .113 P(A | W) = .057 P(M) = .55 P(W) = 1 - .55 = .45

$$P(W\,|\,A) = \frac{P(W)P(A\,|\,W)}{P(W)P(A\,|\,W) + P(M)P(A\,|\,M)} = \frac{(0.45)(.057)}{(0.45)(.057) + (0.55)(.113)} = \frac{0.026}{0.088} = .292$$

## CHAPTER 4 (CONTINUOUS PROBABILITY DISTRIBUTIONS) SOLUTIONS

Use the standard normal distribution table of probabilities on page 139 of your text for all Ch. 4 problems.

**Page 146 # 11)** (a) $P(-1 \leq z \leq 0)$ Look up z-value of 1.00 probability = **.3413**

(b) $P(-1.5 \leq z \leq 0)$ Look up z-value of 1.50 probability = **.4332**

(c) $P(-2 \leq z \leq 0)$ Look up z-value of 2.00 probability = **.4772**

(d) $P(-2.5 \leq z \leq 0)$ Look up z-value of 2.50 probability = **.4938**

(e) $P(-3 \leq z \leq 0)$ Look up z-value of 3.00 probability = **.4986**

**Page 147 # 13)**

(a) $P(-1.98 \leq z \leq .49)$ Look up z-values of 1.98 (probability = .4761) and 0.49 (probability = .1879) and sum the probabilities: .4761 + .1879 = **.664**

(b) $P(.52 \leq z \leq 1.22)$ Since both values are positive, we will subtract the probability for z = .52 from the probability for z = 1.22 . The probability for z = 1.22 is .3888, probability for z = 0.52 is .1985, .3888 - .1985 = **.1903**

(c) $P(-1.75 \leq z \leq -1.04)$ Since both values are negative, we will subtract the probability for z = 1.04 from the probability for z = 1.75. The probability for z = 1.75 is .4599, probability for z = 1.04 is .3508, .4599 - .3508 = **.1091**

**Page 147 # 18)**

(a) $P(X \geq 60 \text{ minutes})$ z-value: z = (60 - 49)/16 = 0.69 Since the problem is a $\geq$ with a positive z-value, determine the probability by taking .5 - z-value.
Look up z-value of 0.69 (prob = .2549) .5 - .2549 = **.2451**

(b) $P(X \leq 30 \text{ minutes})$ Z-value: z = (30 - 49)/16 = -1.19 Since the problem is a $\leq$ with a negative z-value, determine the probability by taking .5 – z-value.
Look up z-value of 1.19 (prob = .3830) .5 - .3830 = **.1170**

(c) Since you want to find the z-value that corresponds to the value for the largest 10% of the curve, find the probability in the table closest to .4000 and determine it’s z-value (for .3997, z-value = 1.28).
To convert the z-value into X, take $\mu + z\sigma$ X = 49 + 1.28(16) = **69.48 minutes**

**Page 147 # 19)**

(a) $P(X \geq \$35,000)$ z-value: z = (35000- 26234)/5000 = 1.75 Since the problem is a $\geq$ with a positive z-value, determine the probability by taking .5 - z-value.
Look up z-value of 1.75 (prob = .4599) .5 - .4599 = **.0401 or 4.01%**

(b) $P(X \leq \$20,000)$ Z-value: z = (20000 - 26234)/5000 = -1.25 Since the problem is a $\leq$ with a negative z-value, determine the probability by taking .5 – z-value.
Look up z-value of 1.25 (prob = .3944) .5 - .3944 = **.1056 or 10.56%**

(c) Since you want to find the z-value that corresponds to the value for the largest 10% of the curve, find the probability in the table closest to .4000 and determine it’s z-value (for .3997, z-value = 1.28).
To convert the z-value into X, take $\mu + z\sigma$ X = 26234 + 1.28(5000) = **$32,634**

# CHAPTER 5 (SAMPLING AND SAMPLING DISTRIBUTIONS) SOLUTIONS

**Page 174 # 11)**

| X | $(X_i\text{-Sample Mean})^2$ |
|---|---|
| 5 | 16 |
| 8 | 1 |
| 10 | 1 |
| 7 | 4 |
| 10 | 1 |
| 14 | 25 |
| 54 | 48 |

(a) $\bar{X} = \frac{\sum X_i}{n} = \frac{54}{6} = 9$

(b) $s^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{48}{5} = 9.6$

$s = \sqrt{9.6} = 3.1$

**Page 175 # 15)**

| X | $(X_i\text{-Sample Mean})^2$ |
|---|---|
| 12.6 | 44.3556 |
| 3.4 | 6.4516 |
| 4.8 | 1.2996 |
| 5.0 | 0.8836 |
| 6.8 | 0.7396 |
| 2.3 | 13.2496 |
| 3.6 | 5.4756 |
| 8.1 | 4.6656 |
| 2.5 | 11.8336 |
| 10.3 | 19.0096 |
| 59.4 | 107.964 |

(a) $\bar{X} = \frac{\sum X_i}{n} = \frac{59.4}{10} = 5.94$

(b) $s^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{107.964}{9} = 11.996$

$s = \sqrt{11.996} = 3.46$

**Page 187 # 18)**

(a) $E(\bar{X}) = \mu = 200$

(b) $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{100}} = \frac{50}{10} = 5$

# CHAPTER 6 (INTERVAL ESTIMATION) SOLUTIONS

**Page 212 # 1)**

(a) $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{40}} = \frac{5}{6.3245} = 0.79$

(b) $z\left(\frac{\sigma}{\sqrt{n}}\right) = z\,(0.79) = 1.96(0.79) = 1.5484$

**Page 212 # 2)**

(a) $\overline{X} \pm z\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 32 \pm 1.645\left(\frac{6}{\sqrt{50}}\right) \Rightarrow 32 \pm 1.645\left(\frac{6}{7.071}\right) \Rightarrow 32 \pm 1.645(0.848)$

$\Rightarrow 32 \pm 1.4 \Rightarrow 30.6 < \mu < 33.4$

We are 90% confident that the population mean is between 30.6 and 33.4.

(b) $\overline{X} \pm z\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 32 \pm 1.96\left(\frac{6}{\sqrt{50}}\right) \Rightarrow 32 \pm 1.96\left(\frac{6}{7.071}\right) \Rightarrow 32 \pm 1.96\,(0.848)$

$\Rightarrow 32 \pm 1.7 \Rightarrow 30.3 < \mu < 33.7$

We are 95% confident that the population mean is between 30.3 and 33.7.

(c) $\overline{X} \pm z\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 32 \pm 2.576\left(\frac{6}{\sqrt{50}}\right) \Rightarrow 32 \pm 2.576\left(\frac{6}{7.071}\right) \Rightarrow 32 \pm 2.576(0.848)$

$\Rightarrow 32 \pm 2.2 \Rightarrow 29.8 < \mu < 34.2$

We are 99% confident that the population mean is between 29.8 and 34.2.

**Page 213 # 6)**

$\overline{X} \pm z\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 369 \pm 1.96\left(\frac{50}{\sqrt{250}}\right) \Rightarrow 369 \pm 1.96\left(\frac{50}{15.811}\right) \Rightarrow 369 \pm 1.96(3.16)$

$\Rightarrow 369 \pm 6.20 \Rightarrow \$362.80 < \mu < \$375.20$

We are 95% confident that the population average weekly earnings for individuals in the service industry is between $362.80 and $375.20.

**Page 213 # 11)**

| X | square of (X-sample mean) |
|---|---|
| 0 | 1.69 |
| 0 | 1.69 |
| 1 | 0.09 |
| 2 | 0.49 |
| 1 | 0.09 |
| 0 | 1.69 |
| 2 | 0.49 |
| 0 | 1.69 |
| 1 | 0.09 |
| 2 | 0.49 |
| 0 | 1.69 |
| 3 | 2.89 |
| 2 | 0.49 |
| 0 | 1.69 |
| 3 | 2.89 |
| 2 | 0.49 |
| 2 | 0.49 |
| 0 | 1.69 |
| 1 | 0.09 |
| 2 | 0.49 |
| 0 | 1.69 |
| 0 | 1.69 |
| 4 | 7.29 |
| 3 | 2.89 |
| 3 | 2.89 |
| 1 | 0.09 |
| 1 | 0.09 |
| 0 | 1.69 |
| 1 | 0.09 |
| 2 | 0.49 |
| 39 | 40.30 |

(a) $$\overline{\mathbf{X}} = \frac{\sum \mathbf{X_i}}{\mathbf{n}} = \frac{\mathbf{39}}{\mathbf{30}} = \mathbf{1.3}$$

(b) $$\mathbf{s^2} = \frac{\sum \left(\mathbf{X_i} - \overline{\mathbf{X}}\right)^2}{\mathbf{n - 1}} = \frac{\mathbf{40.3}}{\mathbf{29}} = \mathbf{1.39}$$

$$\mathbf{s} = \sqrt{\mathbf{1.39}} = \mathbf{1.2}$$

(c) $$\overline{X} \pm z\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 1.3 \pm 1.96\left(\frac{1.2}{\sqrt{30}}\right)$$

$$\Rightarrow 1.3 \pm 1.96\left(\frac{1.2}{5.477}\right) \Rightarrow 1.3 \pm 1.96(0.22)$$

$$\Rightarrow 1.3 \pm 0.43 \Rightarrow 0.87 < \mu < 1.73$$

We are 95% confident that the population average number of defects per car is between 0.87 and 1.73.

(d) A larger sampling of cars will provide us with a smaller confidence interval. If you truly desire to see if the average number of defects is different from the industry average, then take a larger sample. With the current sample, you can say that the number of defects is not statistically different than the industry average.

**Page 219 # 15)**

| X | ($X_i$-Sample Mean)$^2$ |
|---|---|
| 10 | 0 |
| 8 | 4 |
| 12 | 4 |
| 15 | 25 |
| 13 | 9 |
| 11 | 1 |
| 6 | 16 |
| 5 | 25 |
| 80 | 84 |

(a) $$\overline{\mathbf{X}} = \frac{\sum \mathbf{X_i}}{\mathbf{n}} = \frac{\mathbf{80}}{\mathbf{8}} = \mathbf{10}$$

(b) $$\mathbf{s^2} = \frac{\sum \left(\mathbf{X_i} - \overline{\mathbf{X}}\right)^2}{\mathbf{n - 1}} = \frac{\mathbf{84}}{\mathbf{8 - 1}} = \mathbf{12}$$

$$s = \sqrt{12} = 3.46$$

(c) $$\alpha = 1 - \text{confidence level} = 1 - 0.95 = 0.05 \qquad \overline{X} \pm t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right) \Rightarrow 10 \pm t_{0.05/2, 8-1}\left(\frac{3.46}{\sqrt{8}}\right)$$

$$\Rightarrow 10 \pm t_{0.025,7}\left(\frac{3.46}{2.828}\right) \Rightarrow 10 \pm 2.365(1.22) \quad \Rightarrow 10 \pm 2.89 \Rightarrow 7.11 < \mu < 12.89$$

We are 95% confident that the population mean is between 7.11 and 12.89.

**Page 219 # 16)**

(a) $\alpha = 1 - \text{confidence level} = 1 - 0.90 = 0.10 \qquad \overline{X} \pm t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) => 17.25 \pm t_{0.10/2,20-1}\left(\frac{3.3}{\sqrt{20}}\right)$

$$=> 17.25 \pm t_{0.05,19}\left(\frac{3.3}{4.472}\right) => 17.25 \pm 1.729(0.738) \quad => 17.25 \pm 1.28 \quad => \quad 15.97 < \mu < 18.53$$

We are 90% confident that the population mean is between 15.97 and 18.53.

(b) $\alpha = 1 - \text{confidence level} = 1 - 0.95 = 0.05 \qquad \overline{X} \pm t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) => 17.25 \pm t_{0.05/2,20-1}\left(\frac{3.3}{\sqrt{20}}\right)$

$$=> 17.25 \pm t_{0.025,19}\left(\frac{3.3}{4.472}\right) => 17.25 \pm 2.093(0.738) \quad => 17.25 \pm 1.54 \quad => \quad 15.71 < \mu < 18.79$$

We are 95% confident that the population mean is between 15.71 and 18.79.

(c) $\alpha = 1 - \text{confidence level} = 1 - 0.99 = 0.01 \qquad \overline{X} \pm t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) => 17.25 \pm t_{0.01/2,20-1}\left(\frac{3.3}{\sqrt{20}}\right)$

$$=> 17.25 \pm t_{0.005,19}\left(\frac{3.3}{4.472}\right) => 17.25 \pm 2.861(0.738) \quad => 17.25 \pm 2.11 \quad => \quad 15.14 < \mu < 19.36$$

We are 99% confident that the population mean is between 15.14 and 19.36.

**Page 219 # 19)** α = 1 – confidence level = 1 – 0.95 = 0.05 $\quad \overline{\mathbf{X}} = 6.525, \ s = 0.5437, \ n = 20$

$$\overline{X} \pm t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) => 6.525 \pm t_{0.05/2,20-1}\left(\frac{0.5437}{\sqrt{20}}\right) \qquad => 6.525 \pm t_{0.025,19}\left(\frac{0.5437}{4.472}\right)$$

$$=> 6.525 \pm 2.093(0.121575) \qquad => 6.525 \pm 0.254 \quad => \quad 6.271 < \mu < 6.779$$

We are 95% confident that the population mean nonprogram minutes on half-hour, prime-time television shows is between 6.271 and 6.779.

**Page 220 # 22)** α = 1 – confidence level = 1 – 0.95 = 0.05 (a) $\overline{X} = 6.86, \ s = 0.777, \ n = 25$

(b) $\overline{X} \pm t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) => 6.86 \pm t_{0.05/2,25-1}\left(\frac{0.777}{\sqrt{25}}\right) \quad => 6.86 \pm t_{0.025,24}\left(\frac{0.777}{5}\right) => 6.86 \pm 2.064(0.1554)$

$$=> 6.86 \pm 0.32 \quad => \quad 6.54 < \mu < 7.18$$

We are 95% confident that the population mean number of hours of sleep each night is between 6.54 and 7.18.

# CHAPTER 7 (HYPOTHESIS TESTING) SOLUTIONS

**Page 254 # 9)**

(a) Critical Value is $-(z_{0.5 - \alpha}) = -(z_{0.5 - 0.05}) = -(z_{0.45}) = -1.645$ Reject the null hypothesis if T. S. $\leq -1.645$

(b) $T.S. = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{9.46 - 10}{\frac{2}{\sqrt{50}}} = \dfrac{-0.54}{\frac{2}{7.071}} = \dfrac{-0.54}{0.2828} = -1.909$ $-1.909 < -1.645$ Reject $H_0$

**Page 254 # 10)**

(a) Critical Value is $z_{0.5-\alpha} = z_{0.5-0.02} = z_{0.48} = 2.05$ Reject the null hypothesis if T. S. $\geq 2.05$

(b) $T.S. = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{16.5 - 15}{\frac{7}{\sqrt{40}}} = \dfrac{1.5}{\frac{7}{6.325}} = \dfrac{1.5}{1.107} = 1.355$ $1.355 < 2.05$ Fail to reject $H_0$

**Page 254 # 11)** Reject the null hypothesis, if T. S. $\leq -(z_{0.5-\alpha}) = -(z_{0.5-0.05}) = -(z_{0.45}) = -1.645$

(a) $T.S. = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{22 - 25}{\frac{12}{\sqrt{100}}} = \dfrac{-3}{\frac{12}{10}} = \dfrac{-3}{1.2} = -2.5$ $-2.5 < -1.645$ Reject $H_0$

(b) $T.S. = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{24 - 25}{\frac{12}{\sqrt{100}}} = -0.83$ $-0.83 > -1.645$ Fail to reject $H_0$

(c) $\mathbf{T.S.} = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{23.5 - 25}{\frac{12}{\sqrt{100}}} = -1.25$ $-1.25 > -1.645$ Fail to reject $H_0$

(d) $\mathbf{T.S.} = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{22.8 - 25}{\frac{12}{\sqrt{100}}} = -1.833$ $-1.833 < -1.645$ Reject $H_0$

**Page 254 # 15)** Reject the null hypothesis, if T. S. $\leq -(z_{0.5-\alpha}) = -(z_{0.5-0.05}) = -(z_{0.45}) = -1.645$

(a) $\mathbf{T.S.} = \dfrac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{9300 - 10192}{\frac{4500}{\sqrt{100}}} = \dfrac{-892}{450} = -1.98$ $-1.98 < -1.645$ Reject $H_0$

(c) Since $H_0$ was rejected, there is enough statistical evidence to conclude that they sell cars below the average dealer price. The manager could possibly explore the reasons why (types of cars sold, geographic considerations, underpricing cars, etc.)

**Page 261 # 19)**

(a) Reject the null hypothesis, if:

T. S. $\geq z_{0.5-\alpha/2} = z_{0.5-0.025} = z_{0.475} = 1.96$ or T. S. $\leq -(z_{0.5-\alpha/2}) = -(z_{0.5-0.25}) = -(z_{0.475}) = -1.96$

(b) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{11 - 10}{\frac{2.5}{\sqrt{36}}} = \dfrac{1}{0.41667} = 2.4$ $2.4 > 1.96$ Reject $H_0$

**Page 262 # 21)** Reject the null hypothesis, if:

T. S. $\geq z_{0.5-\alpha/2} = z_{0.5-0.025} = z_{0.475} = 1.96$ or T. S. $\leq -(z_{0.5-\alpha/2}) = -(z_{0.5-0.25}) = -(z_{0.475}) = -1.96$

(a) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{22 - 25}{\frac{10}{\sqrt{80}}} = \dfrac{-3}{1.118} = -2.68$ $-2.68 < -1.96$ Reject $H_0$

(b) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{27 - 25}{\frac{10}{\sqrt{80}}} = \dfrac{2}{1.118} = 1.79$ $-1.96 < 1.79 < 1.96$ Fail to reject $H_0$

(c) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{23.5 - 25}{\frac{10}{\sqrt{80}}} = \dfrac{-1.5}{1.118} = -1.34$ $-1.96 < -1.34 < 1.96$ Fail to reject $H_0$

(d) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{28 - 25}{\frac{10}{\sqrt{80}}} = \dfrac{3}{1.118} = 2.68$ $2.68 > 1.96$ Reject $H_0$

**Page 263 # 25)**

(a) Reject the null hypothesis, if:

T. S. $\geq z_{0.5-\alpha/2} = z_{0\cdot 5-0.025} = z_{0.475} = 1.96$ or T. S. $\leq -(z_{0.5-\alpha/2}) = -(z_{0.5-0.025}) = -(z_{0.475}) = -1.96$

(b) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{16.32 - 16}{\frac{0.8}{\sqrt{30}}} = \dfrac{0.32}{0.146} = 2.19$ $2.19 > 1.96$ Reject $H_0$

Shut the production line down.

(c) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{15.82 - 16}{\frac{0.8}{\sqrt{30}}} = \dfrac{-0.18}{0.146} = -1.23$ $-1.96 < -1.23 < 1.96$ Fail to reject $H_0$

Do not shut the production line down.

**Page 266 # 29)**

(a) Reject the null hypothesis, if T. S. $\geq t_{\alpha,\, n-1} = t_{0.05,\, 15} = 1.753$

(b) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{11-10}{\frac{3}{\sqrt{16}}} = \dfrac{1}{0.75} = 1.33$ $1.33 < 1.753$ Fail to reject $H_0$

**Page 267 # 30)** (a) $\overline{X} = \dfrac{\sum X_i}{n} = \dfrac{18+20+16+19+17+18}{6} = \dfrac{108}{6} = 18$

| X | ($X_i$-Sample Mean)$^2$ |
|---|---|
| 16 | 4 |
| 17 | 1 |
| 18 | 0 |
| 18 | 0 |
| 19 | 1 |
| 20 | 4 |
| 108 | 10 |

(b) $s = \sqrt{\dfrac{\sum (x_i - \overline{x})^2}{n-1}} = \sqrt{\dfrac{10}{5}} = 1.414$

(c) Reject the null hypothesis, if
T. S. $\geq t_{\alpha/2,\, n-1} = t_{0.025,\, 5} = 2.571$ or T. S. $\leq -(t_{\alpha/2,\, n-1}) = -(t_{0.025,\, 5}) = -2.571$

(d) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{18-20}{\frac{1.414}{\sqrt{6}}} = \dfrac{-2}{0.577} = -3.46$ (e) $-3.46 < -2.571$ Reject the null hypothesis

**Page 267 # 31)** n=22, s = 18 Reject the null hypothesis, if T. S. $\leq -(t_{\alpha,\, n-1}) = -(t_{0.05,\, 21}) = -1.721$

(a) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{13-15}{\frac{8}{\sqrt{22}}} = \dfrac{-2}{1.706} = -1.17$ $-1.17 > -1.721$ Fail to reject $H_0$

(b) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{11.5-15}{\frac{8}{\sqrt{22}}} = \dfrac{-3.5}{1.706} = -2.05$ $-2.05 < -1.721$ Reject $H_0$

(c) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{15-15}{\frac{8}{\sqrt{22}}} = \dfrac{0}{1.706} = 0$ $0 > -1.721$ Fail to reject $H_0$

(d) $\mathbf{T.S.} = \dfrac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{19-15}{\frac{8}{\sqrt{22}}} = \dfrac{4}{1.706} = 2.34$ $2.34 > -1.721$ Fail to reject $H_0$

**Page 267 # 33**)

(a) $\bar{X} = \frac{\sum X_i}{n} = \frac{4050}{15} = \$270$

(b) s = \$24.98

(c) Step 1: $H_0: \mu \le \$258$
$H_a: \mu > \$258$

Step 2: Level of significance = 0.05, n = 15
Reject the null hypothesis if the Test Statistic is greater than or equal to 1.761.

Step 3: $\bar{X} = \$270,\ s = \$24.78,\ n = 15$

Step 4: $T.S. = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{270 - 258}{\frac{24.78}{\sqrt{15}}} = \frac{12}{6.398} = 1.876$

Step 5: 1.876 > 1.761 Reject the null hypothesis. There is statistical evidence that the population average round trip discount fare has increased in March.

**Page 267 # 34**)

Step 1: $H_0: \mu = \$90$
$H_a: \mu \ne \$90$

Step 2: Level of significance = 0.05, n = 25
Reject the null hypothesis if the Test Statistic is less than or equal to -2.064 or the Test Statistic is greater than or equal to 2.064.

Step 3: $\bar{X} = \$84.50,\ s = \$14.50,\ n = 25$

Step 4: $T.S. = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{84.5 - 90}{\frac{14.5}{\sqrt{25}}} = \frac{-5.5}{2.9} = -1.90$

Step 5: -2.064 < -1.90 < 2.064 Fail to reject the null hypothesis. There is not enough statistical evidence to conclude the population average amount spent per day by U. S. households is different than \$90.

# CHAPTER 9 (DECISION ANALYSIS) SOLUTIONS

**Page 344 # 2**)

(a, b) – Assume values are profits (larger number is better)

| | States of Nature | | | | | |
|---|---|---|---|---|---|---|
| **Alternative** | $s_1$ | $s_2$ | $s_3$ | $s_4$ | Best value | Worst value |
| $d_1$ | 14 | 9 | 10 | 5 | **14** | 5 |
| $d_2$ | 11 | 10 | 8 | 7 | 11 | 7 |
| $d_3$ | 9 | 10 | 10 | 11 | 11 | **9** |
| $d_4$ | 8 | 10 | 11 | 13 | 13 | 8 |

Optimistic: choose best of best values: choose max of (14, 11, 11, 13): choose Alt $d_1$
Conservative: choose best of worst values: choose max of (5, 7, 9, 8): choose Alt $d_3$

(c) Minimax regret:

Step 1: choose best value for each state of nature: $s_1 = 14$, $s_2 = 10$, $s_3 = 11$, $s_4 = 13$

Step 2: Subtract each payoff in a state of nature from the best value for that state of nature to form regret matrix

Step 3: Identify the largest regret for each alternative

| | Regret Matrix | | | | |
|---|---|---|---|---|---|
| **Alternative** | $s_1$ | $s_2$ | $s_3$ | $s_4$ | Largest regret |
| $d_1$ | 14 – 14 = 0 | 10 – 9 = 1 | 11 – 10 = 1 | 13 – 5 = 8 | 8 |
| $d_2$ | 14 – 11 = 3 | 10 – 10 = 0 | 11 – 8 = 3 | 13 – 7 = 6 | 6 |
| $d_3$ | 14 – 9 = 5 | 10 – 10 = 0 | 11 – 10 = 1 | 13 – 11 = 2 | **5** |
| $d_4$ | 14 – 8 = 6 | 10 – 10 = 0 | 11 – 11 = 0 | 13 – 13 = 0 | 6 |

Step 4: Pick the smallest of the largest regrets: min (8, 6, 5, 6): choose Alt $d_3$

(b) Your preferred approach depends on your risk attitude. If you are a risk taker, go with Alt $d_1$. If you are a risk avoider, go with Alt $d_3$. If you are risk neutral, you might go for an option $d_4$.

(c) Assume values are costs (smaller number is better)

| | States of Nature | | | | | |
|---|---|---|---|---|---|---|
| **Alternative** | $s_1$ | $s_2$ | $s_3$ | $s_4$ | Best value | Worst value |
| $d_1$ | 14 | 9 | 10 | 5 | **5** | 14 |
| $d_2$ | 11 | 10 | 8 | 7 | 7 | **11** |
| $d_3$ | 9 | 10 | 10 | 11 | 9 | **11** |
| $d_4$ | 8 | 10 | 11 | 13 | 8 | 13 |

Optimistic: choose best of best values: choose min of (5, 7, 9, 8): choose Alt $d_1$
Conservative: choose best of worst values: choose min of (14, 11, 11, 13): choose Alt $d_2$ or $d_3$

**Page 344 # 2**) part (c) continued

(c) Minimax regret:

Step 1: choose best (smallest) value for each state of nature: $s_1 = 8$, $s_2 = 9$, $s_3 = 8$, $s_4 = 5$

Step 2: Subtract the best value from each payoff in a state of nature to form regret matrix (regrets can **never** be negative)

Step 3: Identify the largest regret for each alternative

| | Regret Matrix | | | | |
|---|---|---|---|---|---|
| **Alternative** | $s_1$ | $s_2$ | $s_3$ | $s_4$ | Largest regret |
| $d_1$ | 14 – 8 = 6 | 9 – 9 = 0 | 10 – 8 = 2 | 5 – 5 = 0 | 6 |
| $d_2$ | 11 – 8 = 3 | 10 – 9 = 1 | 8 – 8 = 0 | 7 – 5 = 2 | **3** |
| $d_3$ | 9 – 8 = 1 | 10 – 9 = 1 | 10 – 8 = 2 | 11 – 5 = 6 | 6 |
| $d_4$ | 8 – 8 = 0 | 10 – 9 = 1 | 11 – 8 = 3 | 13 – 5 = 8 | 8 |

Step 4: Pick the smallest of the largest regrets: min (6, 3, 6, 8): choose Alt $d_2$

**Page 344 # 4**)

(a) The decision is to choose the best lease option; there are three alternatives (Forno Saab, Midtown Motors, and Hopkins Automotive). The chance event is the number of miles Amy will drive per year. There are three possible states of nature associated with this chance event (drive 12K miles/year, drive 15K miles/year, drive 18K miles/year).

(b) The payoff table for Amy's problem is shown below. To illustrate how the payoffs were computed, we show how to compute the total cost of the Forno Saab lease assuming Amy drives 15,000 miles per year (45,000 miles over the 36 month lease).

Total Cost = (Total Monthly Charges) + (Total Additional Mileage Cost)
= 36($299) + $0.15(45,000 - 36,000)
= $10,764 + $1350
= $12,114

(c)

| | States of Nature | | | | |
|---|---|---|---|---|---|
| **Alternative** | 12K/year | 15K/year | 18K/year | Best value | Worst value |
| Forno Saab | $10,764 | $12,114 | $13,464 | **$10,764** | $13,464 |
| Midtown | $11,160 | $11,160 | $12,960 | $11,160 | $12,960 |
| Hopkins | $11,700 | $11,700 | $11,700 | $11,700 | **$11,700** |

Optimistic Approach: Forno Saab ($10,764)
Conservative Approach: Hopkins Automotive ($11,700)

Opportunity Loss or Regret Table

| | Regret Matrix | | | |
|---|---|---|---|---|
| **Alternative** | 12K/year | 15K/year | 18K/year | Largest regret |
| Forno Saab | 0 | $954 | $1,764 | $1764 |
| Midtown | $396 | 0 | $1,260 | $1260 |
| Hopkins | $936 | $540 | 0 | **$936** |

Minimax Regret Approach: Hopkins Automotive

| | | |
|---|---|---|
| (d) | EV (Forno Saab) | 0.5($10,764) + 0.4($12,114) + 0.1($13,464) = $11,574 |
| | EV (Midtown Motors) | 0.5($11,160) + 0.4($11,160) + 0.1($12,960) = $11,340 |
| | EV (Hopkins Automotive) | 0.5($11,700) + 0.4($11,700) + 0.1($11,700) = $11,700 |

Expected Value: Midtown Motors

| | | |
|---|---|---|
| (f) | EV (Forno Saab) | 0.3($10,764) + 0.4($12,114) + 0.3($13,464) = $12,114 |
| | EV (Midtown Motors) | 0.3($11,160) + 0.4($11,160) + 0.3($12,960) = $11,700 |
| | EV (Hopkins Automotive) | 0.3($11,700) + 0.4($11,700) + 0.3($11,700) = $11,700 |

Expected Value: Midtown Motors or Hopkins Automotive

With these probabilities, Amy would be indifferent between the Midtown Motors and Hopkins Automotive leases. However, if the probability of driving 18,000 miles per year goes up any further, the Hopkins Automotive lease will be the best.

**Page 350-351 # 15**)

| | | |
|---|---|---|
| (a) | EV (small center) | 0.1($ 400) + 0.6($ 500) + 0.3($ 660) = $ 538 |
| | EV (medium center) | 0.1(-$ 250) + 0.6($ 650) + 0.3($ 800) = $ 605 |
| | EV (large center) | 0.1(-$ 400) + 0.6($ 580) + 0.3($ 990) = $ 605 |

Based on maximizing expected net cash flow, I would be indifferent to choosing to build a medium or large center.

(d) With the new probabilities of worst case scenario at .2, base case at .5, and the best case at .3:

| | |
|---|---|
| EV (small center) | 0.2($ 400) + 0.5($ 500) + 0.3($ 660) = $ 528 |
| EV (medium center) | 0.2(-$ 250) + 0.5($ 650) + 0.3($ 800) = $ 515 |
| EV (large center) | 0.2(-$ 400) + 0.5($ 580) + 0.3($ 990) = $ 507 |

Based on maximizing expected net cash flow, I would have a slight preference for the small center (although with the expected cash flows separated by $21 between the largest and smallest value, no alternative has a clear cut advantage). A relatively small change in the probabilities caused a change in the preferred alternative.

(e) With the $150,000 expenditure on a promotional campaign and no chance of a worst case scenario:

| | |
|---|---|
| EV (small center) | 0.6($ 500) + 0.4($ 660) - $150= $ 414 |
| EV (medium center) | 0.6($ 650) + 0.4($ 800) - $150= $ 560 |
| EV (large center) | 0.6($ 580) + 0.4($ 990) - $150= $ 594 |

The preferred alternative is a large center with an expected cash flow (which includes the $150,000 expenditure) of $594,000. Since the expected cash flow is less than the cash flow of $605,000 under the original scenario, the promotional campaign may not be worth it. However, you have improved your risk situation by eliminating the possibility of a negative cash flow, so you may want to consider the promotional campaign (it all depends on your attitude toward risk).

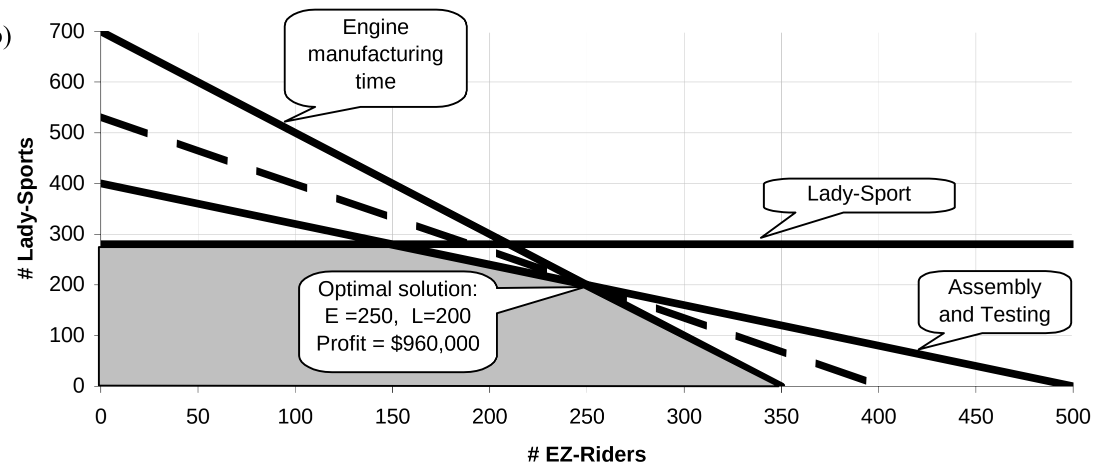
# CHAPTER 11 (INTRO TO LINEAR PROGRAMMING) SOLUTIONS

**Page 419 # 14**)

(a) Let E = number of units of the EZ-Rider produced
L = number of units of the Lady-Sport produced

$$\begin{array}{llrl}
\text{Max} & 2400E + 1800L & & \\
\text{s.t.} & & & \\
 & 6E + 3L \le 2100 & & \text{Engine time} \\
 & L \le 280 & & \text{Lady-Sport maximum} \\
 & 2E + 2.5L \le 1000 & & \text{Assembly and testing} \\
 & E, L \ge 0 & &
\end{array}$$

(b)




(c) The binding constraints are the manufacturing time and the assembly and testing time.

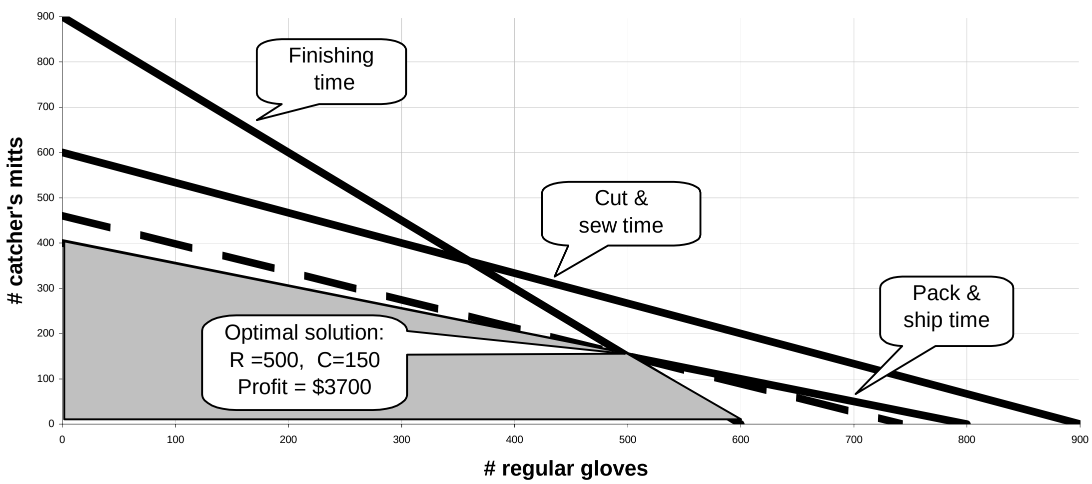
**Page 421 # 22**)

(a) Let R = number of units of regular model.
C = number of units of catcher's model.

$$\begin{array}{llcllll} \text{Max} & 5R & + & 8C & & & \\ \text{s.t.} & & & & & & \\ & 1R & + & \frac{3}{2}C & \le & 900 & \text{Cutting and sewing} \\ & \frac{1}{2}R & + & \frac{1}{3}C & \le & 300 & \text{Finishing} \\ & \frac{1}{8}R & + & \frac{1}{4}C & \le & 100 & \text{Packing and Shipping} \end{array}$$

$R,\ C \ge 0$

(b)




(c) $5(500) + 8(150) = \$3{,}700$

(d)

Cutting & Sewing: $1(500) + \frac{3}{2}(150) = 725$

Finishing: $\frac{1}{2}(500) + \frac{1}{3}(150) = 300$

Packaging & Shipping: $\frac{1}{8}(500) + \frac{1}{4}(150) = 100$

(e)

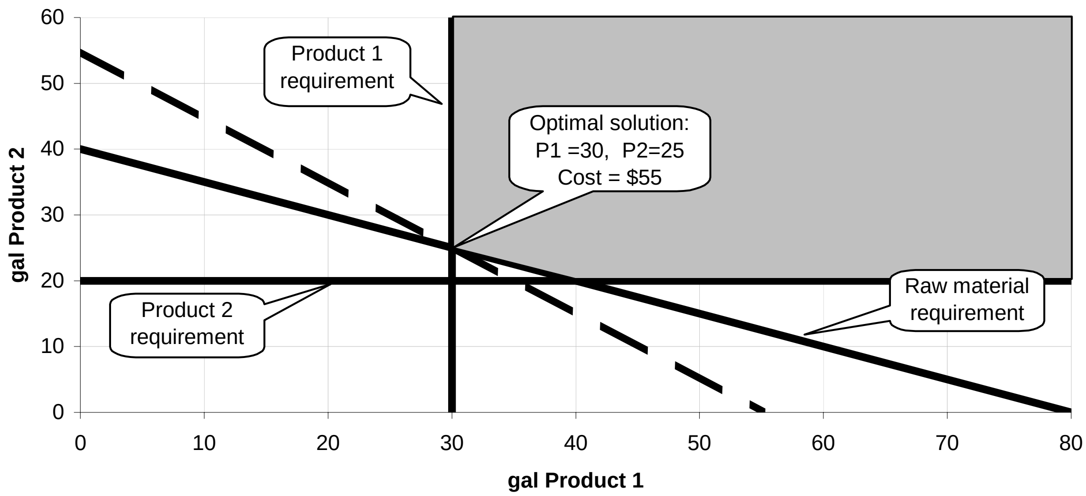
| Department | Capacity | Usage | Slack |
|---|---|---|---|
| Cutting & Sewing | 900 | 725 | 175 hours |
| Finishing | 300 | 300 | 0 hours |
| Packaging & Shipping | 100 | 100 | 0 hours |

**Page 426 # 38**)

Let P1 = gallons of product 1 produced
P2 = gallons of product 2 produced

| | | | | | | |
|---|---|---|---|---|---|---|
| Min | \$1 P1 | + | \$1 P2 | | | |
| s.t. | | | | | | |
| | P1 | | | $\geq$ | 30 | At least 30 gal of Prod 1 |
| | | | P2 | $\geq$ | 20 | At least 20 gal of Prod 2 |
| | 1 P1 | + | 2 P2 | $\geq$ | 80 | At least 80 lbs. of raw mat |

P1, P2 $\geq 0$

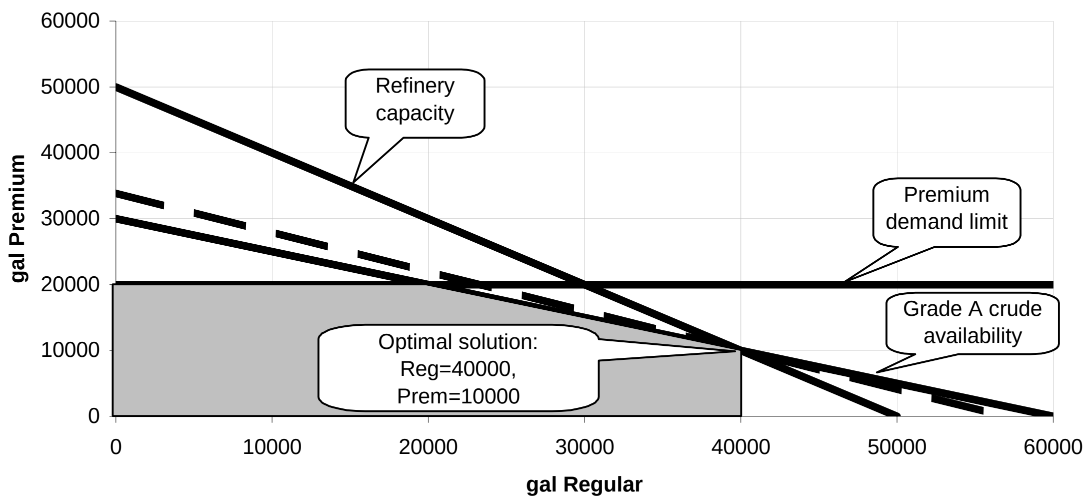


**Page 426 # 39**)

(a) Let R = gallons of regular gasoline produced
P = gallons of premium gasoline produced

$$\begin{aligned} \text{Max} \quad & .30R + .50P \\ \text{s.t.} \quad & \\ & .3R + .6P \leq 18000 \quad \text{Grade A crude limit} \\ & R + P \leq 50000 \quad \text{Refinery limit of 50K gal} \\ & P \leq 20000 \quad \text{Prem dem at most 20K gal} \\ & R, P \geq 0 \end{aligned}$$

(b)




(c) Grade A crude limit: .3(40000) + .6(10000) = 18000 gallons
Refinery capacity: 40000 + 10000 = 50000 gallons
Premium Demand: 10000 gallons used

| Resource | Actual use | Availability | Slack |
|---|---|---|---|
| Grade A crude limit | 18000 gal | 18000 gal | 0 gal |
| Refinery capacity | 50000 gal | 50000 gal | 0 gal |
| Premium Demand | 10000 gal | 20000 gal | 10000 gal |

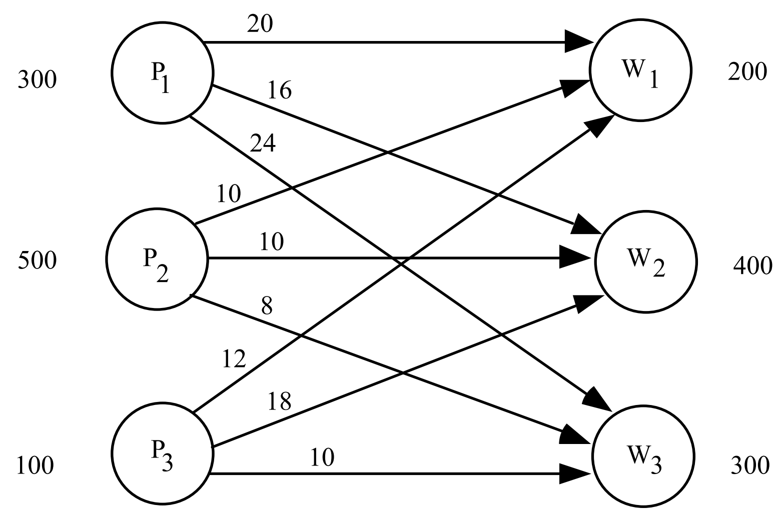
Interpretation of slack variables: All of the Grade A crude is used to produce the optimal mix of gasolines. The refinery is operating at full capacity. There are 10,000 gallons of unmet demand for premium gasoline (it is not profitable for us to meet that demand, because of limits on Grade A crude and refinery capacity).

(d) The binding constraints are Grade A crude and refinery capacity (they have 0 slack). Premium demand is a non-binding constraint (we have slack available).

# CHAPTER 13 (TRANSPORTATION, ASSIGNMENT, AND TRANSSHIPMENT PROBLEMS) SOLUTIONS

**Page 515 # 4**)

(a)

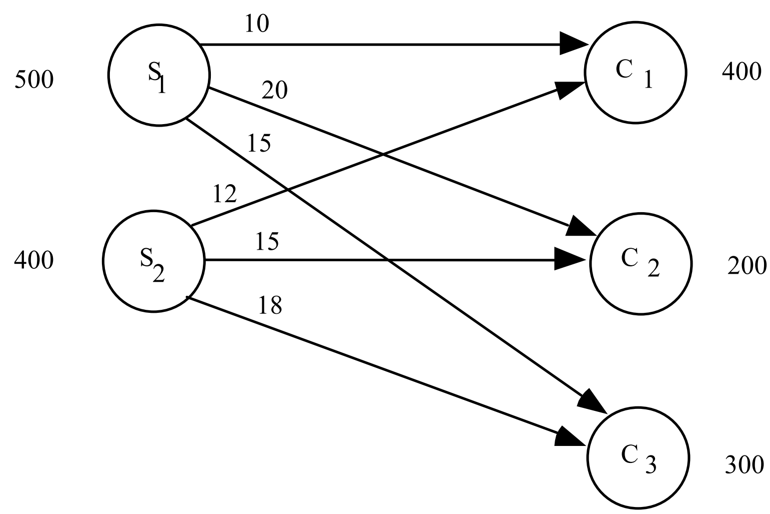



(b) Let $x_{ij}$ = Amount shipped from plant $i$ to warehouse $j$

$$
\begin{array}{llllllllllll}
\text{Min} & 20x_{11} + 16x_{12} + 24x_{13} + 10x_{21} + 10x_{22} + 8x_{23} + 12x_{31} + 18x_{32} + 10x_{33} \\
\text{s.t.} & \\
& x_{11} + x_{12} + x_{13} & \leq 300 \\
& x_{21} + x_{22} + x_{23} & \leq 500 \\
& x_{31} + x_{32} + x_{33} & \leq 100 \\
& x_{11} + x_{21} + x_{31} & = 200 \\
& x_{12} + x_{22} + x_{32} & = 400 \\
& x_{13} + x_{23} + x_{33} & = 300
\end{array}
$$

$$x_{ij} \geq 0 \quad i = 1, 2, 3; \; j = 1, 2, 3$$

**Page 516 # 5**)
(a)



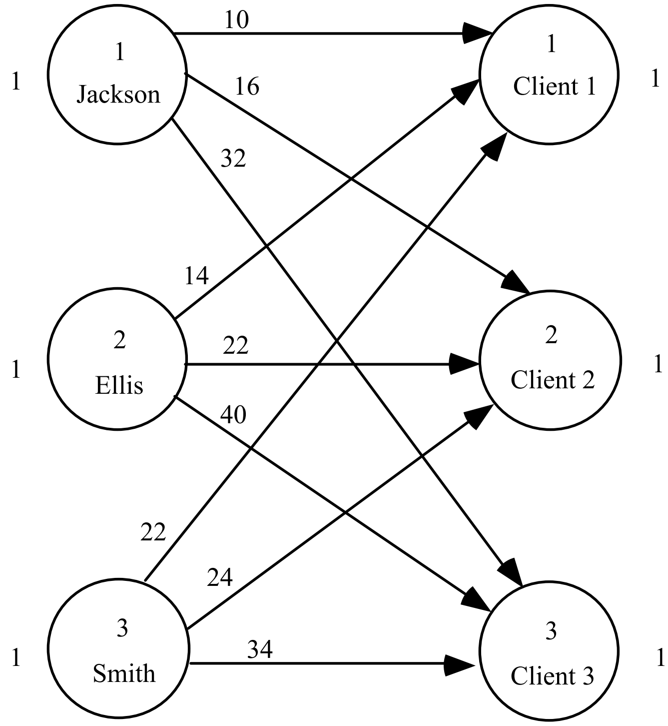

(b) Let $x_{ij}$ = Units of natural gas shipped from supplier *i* to county *j*

$$
\begin{array}{lcccccccccccccl}
\text{Min} & 10x_{11} & + & 20x_{12} & + & 15x_{13} & + & 12x_{21} & + & 15x_{22} & + & 18x_{23} & & \\
\text{s.t.} & & & & & & & & & & & & & \\
& x_{11} & + & x_{12} & + & x_{13} & & & & & & & \leq & 500 \\
& & & & & & & x_{21} & + & x_{22} & + & x_{23} & \leq & 400 \\
& x_{11} & & & & & + & x_{21} & & & & & = & 400 \\
& & & x_{12} & & & & & + & x_{22} & & & = & 200 \\
& & & & & x_{13} & & & & & + & x_{23} & = & 300
\end{array}
$$

$$x_{ij} \geq 0 \quad i = 1, 2; \; j = 1, 2, 3$$

**Page 520 # 12**)

(a)




(a)

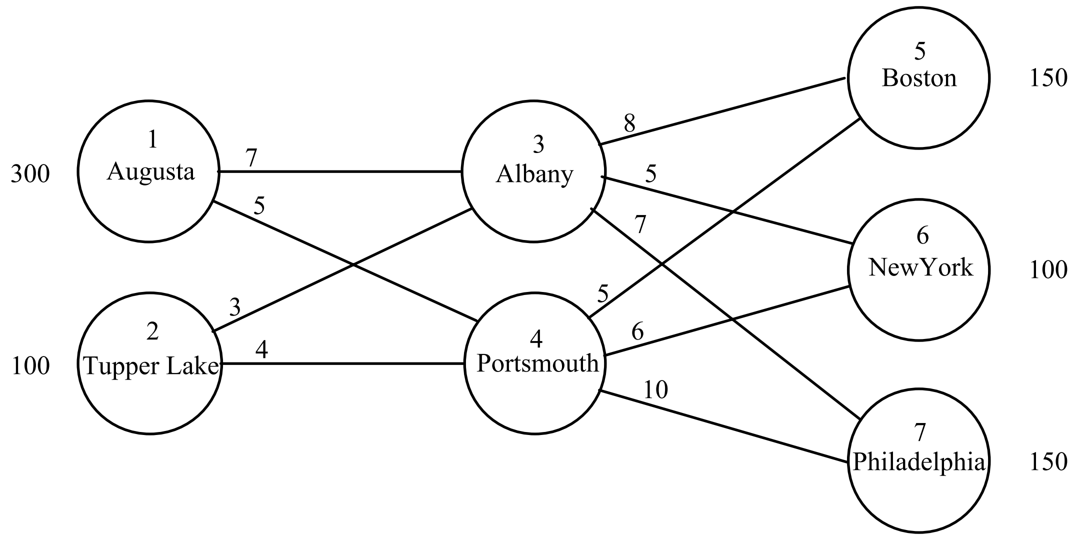
(b) Let $x_{ij} = 1$ if team leader i is assigned to client j (i=1,2,3 j=1,2,3), 0 otherwise

$$\begin{array}{llllllllllll}
\text{Min} & 10x_{11} & + 16x_{12} & + 32x_{13} & + 14x_{21} & + 22x_{22} & + 40x_{23} & + 22x_{31} & + 24x_{32} & + 34x_{33} & & \\
\text{s.t.} & & & & & & & & & & & \\
 & x_{11} & + x_{12} & + x_{13} & & & & & & & \leq & 1 \\
 & & & & x_{21} & + x_{22} & + x_{23} & & & & \leq & 1 \\
 & & & & & & & x_{31} & + x_{32} & + x_{33} & \leq & 1 \\
 & x_{11} & & & + x_{21} & & & + x_{31} & & & = & 1 \\
 & & x_{12} & & & + x_{22} & & & + x_{32} & & = & 1 \\
 & & & x_{13} & & & + x_{23} & & & + x_{33} & = & 1
\end{array}$$

$x_{ij} \geq 0$ for all $i, j$

**Page 525 # 26**)

(a)



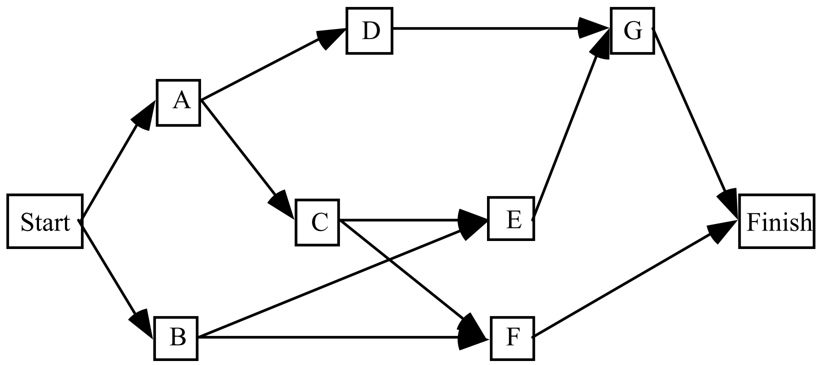

(b)

$$
\begin{array}{ll}
\text{Min} & 7x_{13} + 5x_{14} + 3x_{23} + 4x_{24} + 8x_{35} + 5x_{36} + 7x_{37} + 5x_{45} + 6x_{46} + 10x_{47} \\
\text{s.t.} & \\
& x_{13} + x_{14} \leq 300 \\
& x_{23} + x_{24} \leq 100 \\
& -x_{13} - x_{23} + x_{35} + x_{36} + x_{37} = 0 \\
& - x_{14} - x_{24} + x_{45} + x_{46} + x_{47} = 0 \\
& x_{35} + x_{45} = 150 \\
& x_{36} + x_{46} = 100 \\
& x_{37} + x_{47} = 150
\end{array}
$$

$x_{ij} \geq 0$ for all $i$ and $j$

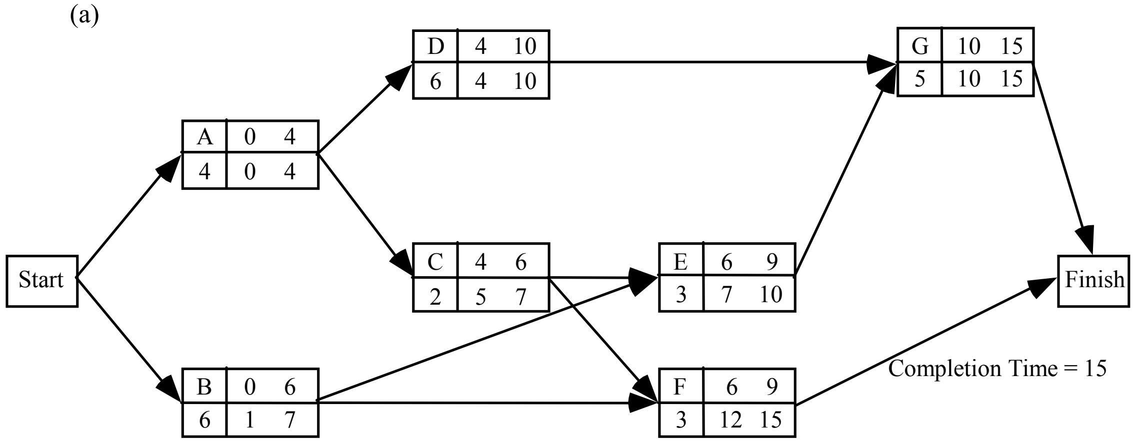
# CHAPTER 14 (PROJECT SCHEDULING: PERT/CPM) SOLUTIONS

**Page 560 # 3)**




**Page 560 # 4)**

(a)

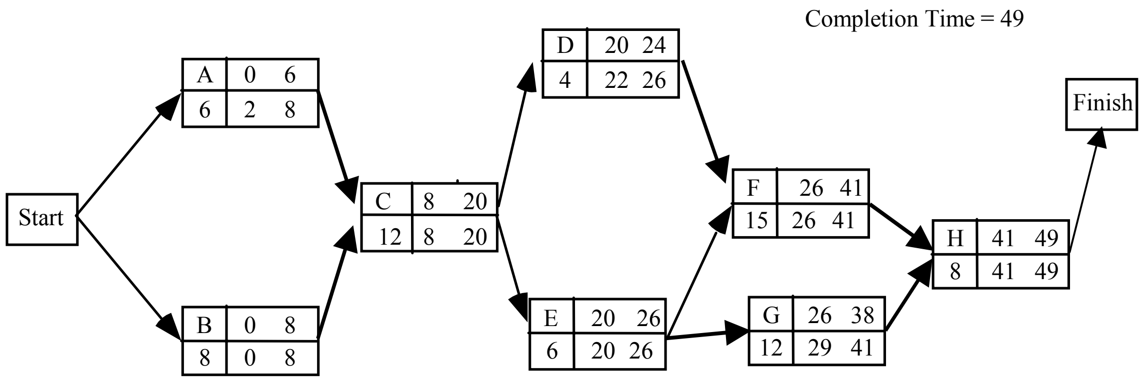



Critical Path: A-D-G (activities where slack = 0, slack =0 where ES=LS)

b. The critical path activities require 15 months to complete. Thus the project should be completed in 1½ years.

**Page 561 # 8**)

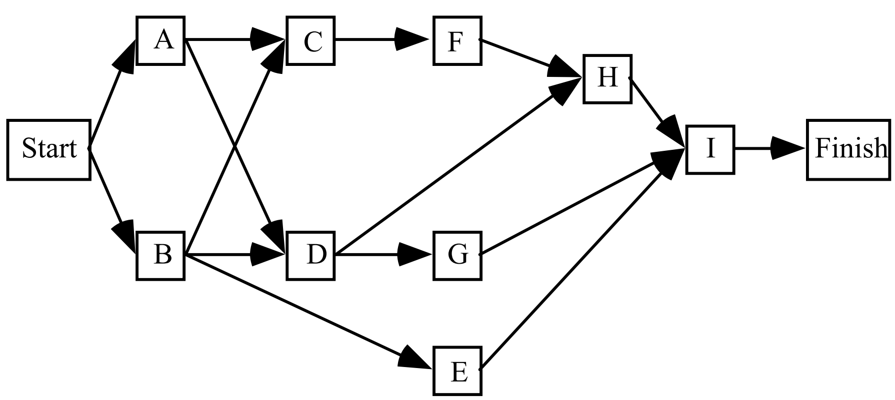
(a)



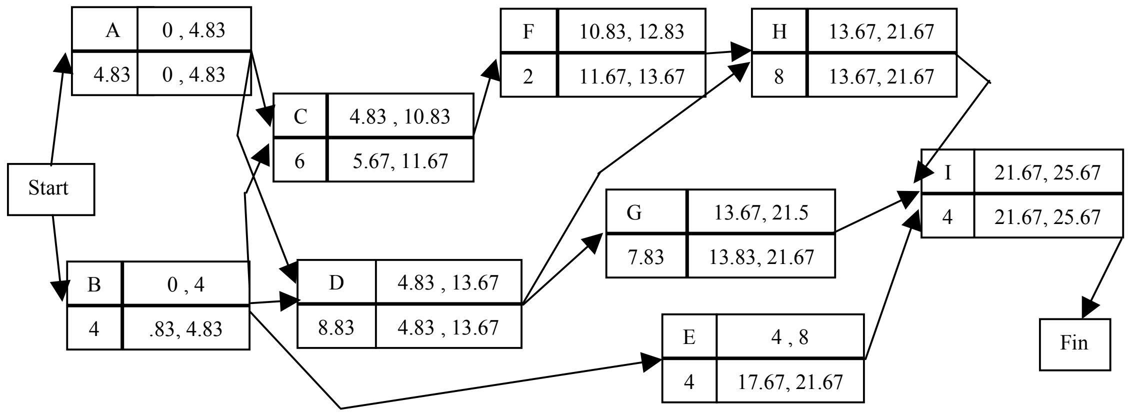

(b) Critical path: B-C-E-F-H (activities where slack = 0, slack =0 where ES=LS)

(d) Since the expected completion time is 49 weeks, construction could begin one year after decision to begin the project. This assumes that the activity times are correct (nothing slips), and the precedence relationships among activities are proper. Any slip in time could cause the project to last longer than one year.

**Page 562 # 11**)



**Page 563 # 12**)




| Activity | Expected Time | Variance |
|---|---|---|
| A | 4.83 | 0.25 |
| B | 4.00 | 0.44 |
| C | 6.00 | 0.11 |
| D | 8.83 | 0.25 |
| E | 4.00 | 0.44 |
| F | 2.00 | 0.11 |
| G | 7.83 | 0.69 |
| H | 8.00 | 0.44 |
| I | 4.00 | 0.11 |

| Activity | Earliest Start | Latest Start | Earliest Finish | Latest Finish | Slack | Critical Activity |
|---|---|---|---|---|---|---|
| A | 0.00 | 0.00 | 4.83 | 4.83 | 0.00 | Yes |
| B | 0.00 | 0.83 | 4.00 | 4.83 | 0.83 | |
| C | 4.83 | 5.67 | 10.83 | 11.67 | 0.83 | |
| D | 4.83 | 4.83 | 13.67 | 13.67 | 0.00 | Yes |
| E | 4.00 | 17.67 | 8.00 | 21.67 | 13.67 | |
| F | 10.83 | 11.67 | 12.83 | 13.67 | 0.83 | |
| G | 13.67 | 13.83 | 21.50 | 21.67 | 0.17 | |
| H | 13.67 | 13.67 | 21.67 | 21.67 | 0.00 | Yes |
| I | 21.67 | 21.67 | 25.67 | 25.67 | 0.00 | Yes |

(a) Critical Path: A-D-H-I

(b) Expected project time: 4.83 + 8.83 + 8 + 4 = 25.66 days (expected times on critical path)

(c) $\sigma^2 = \sigma_A^2 + \sigma_D^2 + \sigma_H^2 + \sigma_I^2 = .25 + .25 + .44 + .11 = 1.05$

Using the normal distribution, $z = \dfrac{25 - E(T)}{\sigma} = \dfrac{25 - 25.66}{\sqrt{1.05}} = -0.65$

From appendix, area for $z$ = -0.65 is 0.2422.

Probability that the project can be completed in 25 days or less = 0.5000 - 0.2422 = 0.2578

# CHAPTER 15 (INVENTORY MANAGEMENT) SOLUTIONS

**Page 608 # 4**)

(a) $Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(12,000)(25)}{(0.20)(2.50)}} = 1095.45 \text{ units}$

(b) r = $r = dm = \frac{1200}{250}(5) = 240 \text{ units}$

(c) $T = \frac{250Q^*}{D} = \frac{250(1095.45)}{12,000} = 22.82 \text{ days}$

(d) Holding cost: ½ Q $C_h$ = ½ (1095.45)(.20)($2.50) = $ 273.86

Ordering cost: (D/Q)$C_o$ = (12000/1095.45)($25) = $ 273.86

Total Cost = $ 547.72

**Page 608 # 5**) Use Q=1000 and calculate the total cost

Holding cost: ½ Q $C_h$ = ½ (1000)(.20)($2.50) = $ 250

Ordering cost: (D/Q)$C_o$ = (12000/1000)($25) = $ 300

Total Cost = $ 550

The total annual inventory cost would increase by only $2.28 (only a .4% increase). Since there is little change in cost, I would probably recommend the policy favored by management.

Reorder point: $r = dm = \frac{1200}{250}(5) = 240$ units (since demand, lead time, and number of working days do not change, the reorder point would stay the same).

**Page 610 # 15**)

(a) $Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h + C_b}{C_b}\right)} = \sqrt{\frac{2(1200)(25)}{0.50}\left(\frac{0.50+5}{0.50}\right)} = 1148.91$

(b) $S^* = Q^*\left(\frac{C_h}{C_h + C_b}\right) = 1148.91\left(\frac{0.50}{0.50+5}\right) = 104.45$

(c) Max inventory = $Q^*$ - $S^*$ = 1044.46

(d) $T = \frac{250Q^*}{D} = \frac{250(1148.91)}{12000} = 23.94$

(e) Holding: $\frac{(Q-S)^2}{2Q}C_h = \$237.38$

Ordering: $\frac{D}{Q}C_o = 261.12$

Backorder: $\frac{S^2}{2Q}C_b = 23.74$

Total Cost: $522.24

The total cost for the EOQ model in problem 4 was $547.72. Allowing backorders reduces the total cost.

**Page 610 # 17**) Using EOQ model: $Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(800)(150}{3}} = 283$

Holding cost: ½ Q $C_h$ = ½ (283)($3) = $ 424.50, Ordering cost: (D/Q)$C_o$ = (800/283)($150) = $ 424.03

Total Cost = $ 848.53

Using backorder model: $Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h + C_b}{C_b}\right)} = \sqrt{\frac{2(800)(150)}{3}\left(\frac{3+20}{20}\right)} = 303$

$$S^* = Q^*\left(\frac{C_h}{C_h + C_b}\right) = 303\left(\frac{3}{20+3}\right) = 40$$

Holding cost: $C_h(Q-S)^2/2Q$ = ($3)(303-40)$^2$/2(303) = $ 342.42,
Ordering cost: (D/Q)$C_o$ = (800/303)($150) = $ 396.04
Backorder cost: $C_bS^2/2Q$ = ($20)(40$^2$)/(2)(303) = $ 52.81

Total cost: $791.27 You can save $848.53 - $791.27 = $57.26 by using a backorder model

# MULTI-ATTRIBUTE DECISION ANALYSIS

In the Decision Analysis chapter of the textbook, the decision analysis techniques discussed have concentrated on what is referred to as single attribute decision analysis. In decision analysis, an attribute is a characteristic of interest associated with an alternative. In virtually all decisions of consequence, more than one attribute is present. In considering the acquisition of a helicopter, for instance, the Army might consider cost, payload, speed, maneuverability, reliability, maintainability, range, fuel consumption and a variety of other things. Systematically considering all of these different characteristics is a difficult task, but techniques do exist to assist the decision-maker in handling multi-attribute decisions. Some of the simpler techniques will be reviewed in this section.

When confronted with a multi-attribute decision situation, the first feature that poses difficulties is the sheer volume of information that must be digested. Assume the army is trying to select between four competing "off the shelf" fixed wing aircraft for use as utility transports. Among the attributes that might be of interest are cost, payload, number of seats, seating configurations possible, speed, range, reliability, maintainability, and fuel consumption. If this list includes all the things the Army might consider (and it may not) the analyst will be confronted with four aircraft, each described by information on nine attributes. That results in thirty-six pieces of information that somehow must be interpreted rationally. Consider further the problem faced by a promotion board reviewing several thousand individuals, each described with respect to several attributes (command time, civilian education, military education, OER's, etc.). The board has to review thousands of individual pieces of information and rank all the officers considered in some logical fashion. In situations like these the volume of the problem is significant.

The problem of volume in multi-attribute decisions is compounded by difficulties with the way the data is recorded. At the most basic level, some of the attributes may be described in a quantitative manner and some in qualitative terms. For example, a certain piece of equipment may induce considerable operator fatigue. The Army might want to compare fatigue levels between alternative brands of that equipment, but the only information available on fatigue might be descriptions like "severe", or "moderate" fatigue induced. Assessing such descriptions is difficult, but still may be useful in making a reasoned decision.

Problems also arise when dealing with quantifiable attributes. Different quantifiable attributes are usually described in different units of measure. Cost is measured in dollars, payload in pounds, mean time between failure in operating hours and so on. How does one form an overall opinion of the value of an alternative by considering a group of attributes described with different units of measure? To complicate things further, some quantitative attribute measures represent more desirable situations when their values are high. Payload and mean time between failure are such attributes. Other attributes, such as cost, are more desirable when their values are low.

A final difficulty in multi-attribute decisions involves comparing the importance of the different attributes. Some attributes to be considered in a decision are clearly more important then others. Determining the exact order of importance and the magnitude of the differences in importance is a difficult task. How does the maneuverability of an aircraft compare in importance with its payload? If maneuverability is more important, how much more important is it? Twice as important? One and a half times as important? Who decides? Logistics planners, interested in moving men and equipment, may have a decidedly different view than the pilots who want to get back in one piece. Obviously, the weighting of attributes can be a highly subjective matter.

In approaching a multi-attribute problem, one of the first things to attempt is a reduction in the amount of data that must be considered. This can be accomplished by reducing the number of attributes being examined, or by eliminating alternatives from consideration. Usually, a reduction in the number of attributes is difficult, but it is still worthwhile to think carefully about which attributes should be closely evaluated. Sometimes, attributes that at first glance may appear important really represent minor characteristics that can be ignored. Of course, this is a subjective judgment that needs to be made with care, but at times it is appropriate.

Reduction in the number of alternatives under consideration can often be made by using one of three simple multi-attribute techniques available: satisficing, lexicography, and dominance. These techniques may not all be appropriate in any given situation, and their use usually does not result in the selection of the best alternative. Their main value lies in narrowing the choices by eliminating alternatives, thus reducing the amount of information to be considered.

**Satisficing** can be applied when there is a clear requirement that must be met by each alternative. For instance, in reviewing new tank recovery vehicles, the Army could specify that an acceptable candidate must be able to tow a fully loaded M-1 over a certain type of terrain. Any candidate vehicle that could not perform this basic task would be eliminated from further consideration.

**Lexicography** can be applied when there is a significant difference in attribute importance. In some circumstances, one attribute might be so much more important than any other attribute that a decision can be based on that attribute alone. For example, in certain procurement actions, cost is the predominant selection criterion. The low cost bidder receives the contract. Such a selection would be regarded as lexicographic. If two or more bidders submitted the same bid, the lexicographic method would then compare the bidders on the second most important attribute, such as delivery schedule. This process would be continued until a final selection was made, comparisons had been made on all attributes, or comparisons had moved into an area where a clear importance ordering of attributes was no longer possible. Lexicography differs very little from single attribute decision making. It is a multi-attribute technique only in the sense that it provides a process to take other attributes into account systematically in the event of a tie when comparing important attributes.

After the number of alternatives have been reduced using satisficing and lexicography, as appropriate, **dominance** can be used for further reduction. Dominance involves comparing each alternative to all the other alternatives under consideration and looking for a situation where one alternative is equal to or better than another across all the attributes. In this case, the first alternative is said to dominate the second alternative, and the second alternative is discarded. This can be very useful in reducing the number of alternatives to be considered, and in some cases may result in a final decision.

Once the number of alternatives has been reduced, techniques can be applied to simplify the dimensional and weighting problems and to rank the alternatives in an order that reflects the decision-maker's preferences. To illustrate these techniques, assume the Army is considering the purchase of a light tactical vehicle. Presume further that there are three vehicles under review. The characteristics of these vehicles appear in Figure 1. Notice that each attribute is indicated as either a cost or a benefit. A cost attribute indicates that a smaller attribute value is better, while a benefit attribute indicates that a larger attribute value is better.

| | Range (miles) **BEN** | Payload (lbs.) **BEN** | Weight (lbs.) **COST** | Reliability **BEN** | Mobility **BEN** | Maintenance Time **COST** |
|---|---|---|---|---|---|---|
| Alt. A | 400 | 2500 | 7500 | High | Average | Average |
| Alt. B | 320 | 2000 | 6600 | Low | Low | High |
| Alt. C | 240 | 1500 | 6000 | Average | High | Very low |

**Figure 1: Vehicle Purchase Problem Characteristics**

The first problem encountered dealing with the data in Figure 25 is that a portion of it is qualitative in nature. It would be helpful if the data concerning reliability, mobility, and maintenance time could be somehow quantified. One of the ways to do this is to ask the decision-maker to rate the verbal descriptions in Figure 1 on a numerical scale. In Figure 2, a scale is presented ranking the descriptors from zero to one, with one being the highest value.

| <u>Cost Attribute</u> | | <u>Benefit Attribute</u> |
|---|---|---|
| Very Low | - 1.00 - | Very High |
| Low | - 0.75 - | High |
| Average | - 0.50 - | Average |
| High | - 0.25 - | Low |
| Very High | - 0.00 - | Very Low |

**Figure 2: Scales for Qualitative Attributes**

Assume further that this scale is valid for all three qualitative attributes. In reality, this may not be the case, and the decisionmaker might construct a different scale for each of the three. Using this scale, it is possible to assign numerical values to the adjectives that appeared in Figure 2. This has been done in Figure 3. This creates a situation where all the attributes can be evaluated mathematically. Bear in mind that many decision-makers will be uncomfortable with scaling information in this manner. If their discomfort is profound enough, the technique should not be used. If the decision-maker has no faith in the technique, he will pay no attention to the recommendations that result. Notice that maintenance time is now listed as a benefit, since the scale now reflects that a higher number is better (alternative C has the best maintenance time).

| | Range (miles) **BEN** | Payload (lbs.) **BEN** | Weight (lbs.) **COST** | Reliability **BEN** | Mobility **BEN** | Maintenance Time **BEN** |
|---|---|---|---|---|---|---|
| Alt. A | 400 | 2500 | 7500 | 0.75 | 0.50 | 0.50 |
| Alt. B | 360 | 2000 | 6600 | 0.25 | 0.25 | 0.25 |
| Alt. C | 240 | 1800 | 6000 | 0.50 | 0.75 | 1.00 |

**Figure 3: Vehicle Purchase Problem - Qualitative Attributes Scaled**

Once the attribute data is in numerical form, it is possible to address the problem of dimensionality. Although the information on reliability, mobility, and maintenance time is now expressed in dimensionless units, the information on the first three attributes is still expressed in miles and pounds. This data can be reduced to the same type of zero to one scale that was used for the non-quantified attributes. Here, however, the transformation can be made numerically, without having to resort to questioning the decision-maker, by using the following formula:

If the attribute is scaled as a **benefit**, divide each attribute value by the **largest** attribute value.

If the attribute is scaled as a **cost**, divide the **smallest** attribute value by each attribute value.

Using the formula above, it is possible to scale the attributes for all alternatives as follows:

**RANGE**: Alt A: 400/400 =1.00, Alt B: 360/400 =.90, Alt C: 240/400 = 0.60
**PAYLOAD**: Alt A: 2500/2500 =1.00, Alt B: 2000/2500 =.80, Alt C: 1800/2500 = 0.72
**WEIGHT**: Alt A: 6000/7500 =.80, Alt B: 6000/6600 =.91, Alt C: 6000/6000 =1.00
**RELIABILITY**: Alt A: .75/.75 =1.00, Alt B: .25/.75 =.33, Alt C: .50/.75 =.67
**MOBILITY**: Alt A: .50/.75 =.67, Alt B: .25/.75 =.33, Alt C: .75/.75 =1.00
**MAINT TIME**: Alt A: .50/1.00 = .50, Alt B: .25/1.00 =.25, Alt C: 1.00/1.00 =1.00

Figure 4 shows the decision matrix will all values rescaled.

| | Range) | Payload | Weight | Reliability | Mobility | Maintenance Time |
|---|---|---|---|---|---|---|
| Alt. A | 1.00 | 1.00 | 0.80 | 1.00 | 0.67 | 0.50 |
| Alt. B | 0.90 | 0.80 | 0.91 | 0.33 | 0.33 | 0.25 |
| Alt. C | 0.60 | 0.72 | 1.00 | 0.67 | 1.00 | 1.00 |

**Figure 4: Vehicle Purchase Problem - All Attributes Scaled**

The data in Figure 4 represents the valuations of all the attributes on a dimensionless scale with a range of zero to one. The qualitative attributes were placed on this scale subjectively by the decision-maker. The quantitative attributes were placed via the linear transformation performed with the formula noted above. These actions have removed the problem of dealing with data expressed in different units of measure. What has yet to be addressed is the relative importance of the six attributes.

As noted above, the comparative importance of these attributes is likely to be highly subjective. Different people may have different ideas about importance. The most appropriate person to provide guidance on this issue is the decision-maker. Perhaps the easiest way to get an idea of his preferences in this matter is to ask him to weight the attributes for importance, assigning them fractional weights that add to one. Assume the decision-maker assigned the following weights:

| | |
|---|---|
| Range = | 0.30 |
| Payload = | 0.15 |
| Weight = | 0.10 |
| Reliability = | 0.20 |
| Mobility = | 0.20 |
| Maintenance Time = | 0.05 |

With these weights, it is possible to calculate weighted average of the scaled attribute values to aid the decision.

**ALT A:** 1.00(.3) + 1.00(.15) + 0.80(.1) + 1.00(.2) + 0.67(.2) + 0.50(.05) = 0.889

**ALT B:** 0.90(.3) + 0.80(.15) + 0.91(.1) + 0.33(.2) + 0.33(.2) + 0.25(.05) = 0.6255

**ALT C:** 0.60(.3) + 0.72(.15) + 1.00(.1) + 0.67(.2) + 1.00(.2) + 1.00(.05) = 0.772

Based on the weights assumed by the decision-maker for each attribute, Alternative A appears to be the best choice.

Whatever the choice of techniques in a decision analysis, it seems clear that this choice should be appealing to the decisionmaker. He should be satisfied that the model is a good one in terms of basic problem structure, method of analysis, estimation techniques, and inclusion of proper priorities. Such considerations make it imperative that an analysis be tailored to the decision-maker. This is precisely what has been attempted in this discussion. There are many problems that can occur. Pinpointing the decision-maker's priorities or attitudes toward risk, and obtaining subjective estimates are not easy problems to overcome. Frequently, the estimates come from a group of experts, and the experts cannot agree on a single estimate. Worse yet, sometimes no one can determine who the decision-maker is. This is not uncommon. All of these problems and more will face a decision analyst.

# MULTI-ATTRIBUTE DECISION ANALYSIS PRACTICAL EXERCISE

**Situation:**

The Army must select the "best" of seven different helicopter systems. The following matrix shows the seven alternatives and the attributes important in making the decision:

| HELICOPTER SELECTION | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | ***ATTRIBUTES*** | | | | | | | |
| | ***Cruise Speed*** | ***Climb Rate*** | ***Endur-ance*** | ***Payload*** | ***Cost*** | ***Surviv-ability*** | ***Maneu-verability*** | ***Reli-ability*** |
| **A1** | 145 | 580 | 1.9 | 2625 | 3.5 | Very High | High | High |
| **A2** | 175 | 415 | 2.1 | 2750 | 4.9 | Low | Average | Average |
| **A3** | 190 | 500 | 2.2 | 2700 | 3.0 | Average | Low | High |
| **A4** | 150 | 450 | 1.8 | 2550 | 2.5 | Very High | Very High | Average |
| **A5** | 140 | 425 | 2.6 | 2500 | 5.1 | High | High | Very High |
| **A6** | 135 | 620 | 2.5 | 2700 | 4.5 | Low | Average | Average |
| **A7** | 170 | 430 | 2.0 | 2600 | 4.0 | High | Average | Low |
| | Ben | Ben | Ben | Ben | Cost | Ben | Ben | Ben |

**Requirement:**

1. Can you eliminate any of the alternatives using ***dominance***? Justify your answer. If you eliminated any of the seven alternatives do not consider them throughout the rest of the problem.

2. The system must meet the following minimum requirements:

| HELICOPTER SELECTION MINIMUM STANDARDS | |
|---|---|
| **Attribute** | **Minimum Requirement** |
| Speed | ≥ 125 |
| Climb Rate | ≥ 425 |
| Endurance | ≥ 1.5 |
| Payload | ≥ 2500 |
| Cost | ≤ 5.0 |
| Survivability | ≥ low |
| Maneuverability | ≥ low |
| Reliability | ≥ low |

Can you eliminate any of the alternatives using ***satisficing***? If so, identify them and explain why you can eliminate them. Do not consider them throughout the rest of the problem.

3. Given the following attribute weights and using ***simple additive weighting*** which alternative would you select and why?

| HELICOPTER SELECTION ATTRIBUTE WEIGHTS | |
|---|---|
| **Attribute** | **Weight** |
| Speed | 10 |
| Climb Rate | 1 |
| Endurance | 18 |
| Payload | 4 |
| Cost | 2 |
| Survivability | 30 |
| Maneuverability | 15 |
| Reliability | 20 |

# MULTI-ATTRIBUTE DECISION ANALYSIS PRACTICAL EXERCISE SOLUTION

**Situation:**

The Army must select the "best" of seven different helicopter systems. The following matrix shows the seven alternatives and the attributes important in making the decision:

| | ***ATTRIBUTES*** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | ***Cruise Speed*** | ***Climb Rate*** | ***Endur-ance*** | ***Payload*** | ***Cost*** | ***Surviv-ability*** | ***Maneu-verability*** | ***Reli-ability*** |
| **A1** | 145 | 580 | 1.9 | 2625 | 3.5 | Very High | High | High |
| **A2** | 175 | 415 | 2.1 | 2750 | 4.9 | Low | Average | Average |
| **A3** | 190 | 500 | 2.2 | 2700 | 3.0 | Average | Low | High |
| **A4** | 150 | 450 | 1.8 | 2550 | 2.5 | Very High | Very High | Average |
| **A5** | 140 | 425 | 2.6 | 2500 | 5.1 | High | High | Very High |
| **A6** | 135 | 620 | 2.5 | 2700 | 4.5 | Low | Average | Average |
| **A7** | 170 | 430 | 2.0 | 2600 | 4.0 | High | Average | Low |
| | Ben | Ben | Ben | Ben | Cost | Ben | Ben | Ben |

**Requirement:**

1. Can you eliminate any of the alternatives using ***dominance***? Justify your answer. If you eliminated any of the seven alternatives do not consider them throughout the rest of the problem.

**No alternative can be eliminated using dominance because no alternative is better than or equal to another alternative for all attributes.**

2. The system must meet the following minimum requirements:

| **HELICOPTER SELECTION MINIMUM STANDARDS** | |
|---|---|
| **Attribute** | **Minimum Requirement** |
| Speed | ≥ 125 |
| Climb Rate | ≥ 425 |
| Endurance | ≥ 1.5 |
| Payload | ≥ 2500 |
| Cost | ≤ 5.0 |
| Survivability | ≥ low |
| Maneuverability | ≥ low |
| Reliability | ≥ low |

Can you eliminate any of the alternatives using ***satisficing***? If so, identify them and explain why you can eliminate them. Do not consider them throughout the rest of the problem.

**A2 has an unsatisfactory climb rate and the cost of A5 is too high. Therefore, these can be eliminated using satisficing.**

3. Given the following attribute weights and using ***simple additive weighting*** which alternative would you select and why?

| **HELICOPTER SELECTION ATTRIBUTE WEIGHTS** | |
|---|---|
| **Attribute** | **Weight** |
| Speed | 10 |
| Climb Rate | 1 |
| Endurance | 18 |
| Payload | 4 |
| Cost | 2 |
| Survivability | 30 |
| Maneuverability | 15 |
| Reliability | 20 |

First, transform the qualitative data using the method outlined in part 3 above. Then normalize the quantitative data using linear proportional scaling. The normalized decision matrix and attribute weights are shown below.

| **HELICOPTER SELECTION NORMALIZED DECISION MATRIX** ***ATTRIBUTES*** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | ***Cruise Speed*** | ***Climb Rate*** | ***Endur-ance*** | ***Payload*** | ***Cost*** | ***Surviv-ability*** | ***Maneu-verability*** | ***Reli-ability*** |
| **Weight** | .10 | .01 | .18 | .04 | .02 | .30 | .15 | .20 |
| **A1** | 0.763 | 0.935 | 0.760 | 0.972 | 0.714 | 1.000 | 0.778 | 1.000 |
| **A3** | 1.000 | 0.806 | 0.880 | 1.000 | 0.833 | 0.556 | 0.333 | 1.000 |
| **A4** | 0.789 | 0.726 | 0.720 | 0.944 | 1.000 | 1.000 | 1.000 | 0.714 |
| **A6** | 0.711 | 1.000 | 1.000 | 1.000 | 0.555 | 0.333 | 0.556 | 0.714 |
| **A7** | 0.895 | 0.694 | 0.800 | 0.963 | 0.625 | 0.778 | 0.556 | 0.429 |

A simple additive weighting score is then calculated for each alternative by multiplying each attribute value by its respective attribute weight and then adding these weighted attribute values together. For example the score for A1 would be found as follows:

A1 = .1(.763) + .01(.935) + .18(.76) + .04(.972) + .02(.714) + .3(1) + .15(.778) + .2(1)
= 0.892.

**The simple additive weighting scores are:**

**A1 = 0.892**
**A3 = 0.740**
**A4 = 0.866**
**A6 = 0.638**
**A7 = 0.694**

**Select A1 since it has the highest score.**